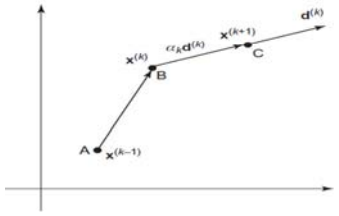
Lecture 07: Methods for One-dimensional Unconstrained Optimization Problems

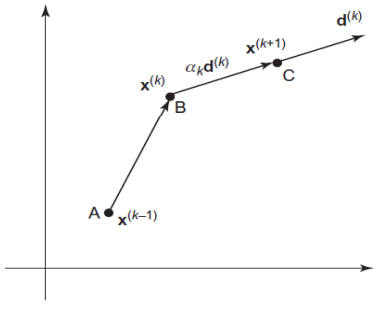


Unconstrained Optimization

Unconstrained optimization

One-dimensional or line search problems	Multidimensional problems
To find a scalar α^* to minimize a function $f(\alpha)$	To find points $\mathbf{x}^*$ to minimize a function $f(\mathbf{x}) = f(x_1, x_2, \ldots x_n)$

Iterative Algorithm



Numerical Methods

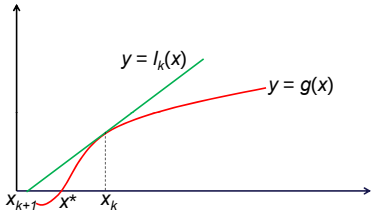
- To find out local minimizer, just find where $f'(x) = 0$.
- But f' may not exist, or may be difficult to calculate, or $f'(x) = 0$ may not be explicitly solvable for x.
- Three types of methods:
  - (A) Solve $f'(x) = 0$: methods to solve $g(x) = 0$, use $g = f'$.
  - (B) Find minimizer of f without using f'.
  - (C) Use information from both f and f'.

Numerical Methods

- (A) methods to solve $g(x) = 0$
  - Newton's method
  - Bisection method
  - Regula falsi (false position) method
  - Secant method
- (B) methods using f and not f'
  - Golden section method
  - Quadratic interpolation method
- (C) methods using both f and f'
  - Cubic interpolation method

Newton's Method

- Given x_k, approximate $g(x)$ by tangent line at x_k.



$$y = l_k(x) = g(x_k) + g'(x_k)\,(x - x_k)$$

Newton's Method

$$y = l_k(x) = g(x_k) + g'(x_k)\,(x - x_k)$$

- Let x_{k+1}, solve $l_k(x)$:

$$0 = g(x_k) + g'(x_k)\,(x_{k+1} - x_k)$$

$$\boxed{x_{k+1} = x_k - g(x_k)/\,g'(x_k)}$$

Newton's Method

- Termination Criteria:
  - (i) $|g(x_k)| < \varepsilon$
  - (ii) $|x_{k+1} - x_k| < \delta$
  - (iii) $k > K$, $|g(x_k)| > E$, $|x_{k+1} - x_k| > D$ → Check for divergence
  - (iv) some combination of the above
- Cons: May diverge, requires g' (f').
- Pros: If converges, very fast convergence.

Newton's Method

Example: find the maximum value of $f(x) = 2\sin x - \frac{x^2}{10}$ with an initial guess of $x_0 = 2.5$.