ME265: Thermal Engineering & Heat Transfer

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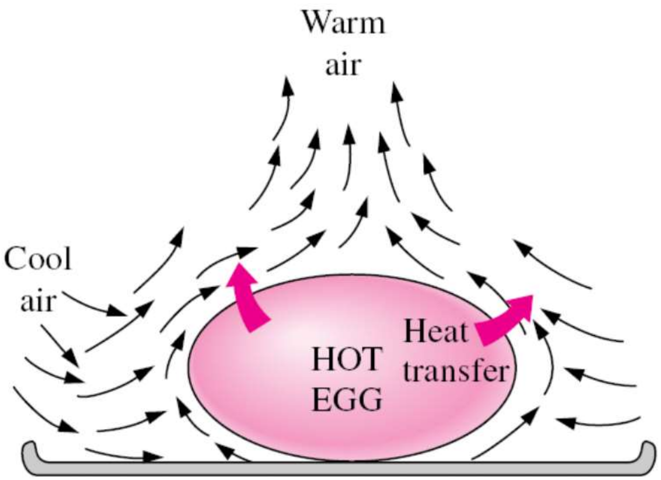
4.3.4 Natural Convection

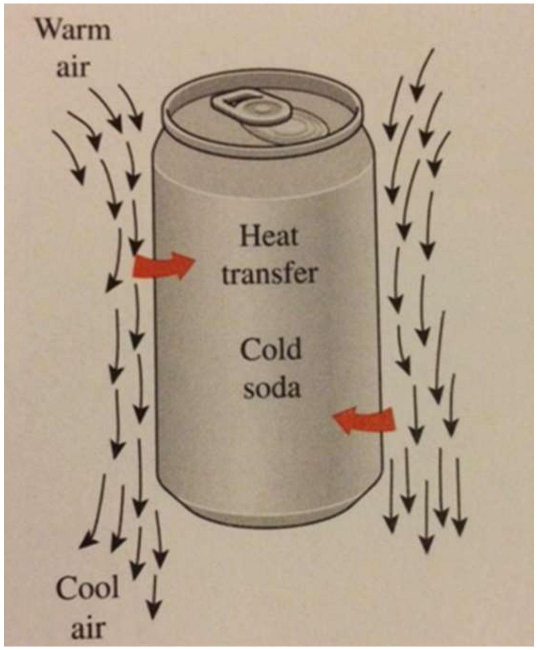
❑ Section objectives:

- Understand the **physical mechanism** of natural convection and **significant dimensionless number**.
- Estimate natural convection coefficient using **empirical correlations**
- Examine natural convection from finned surfaces, and **determining the optimum fin spacing**.
- Analyze **natural convection inside enclosures** such as double-pane windows.

4.3.4 Natural Convection

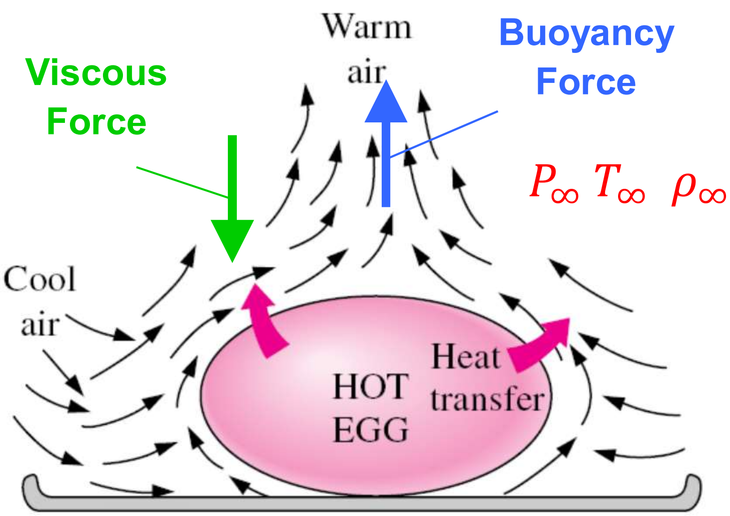


The motion that results from the continual replacement of the heated air in the vicinity of the hot body by the cooler air nearby is called a **natural convection current.**



The motion that results from the continual replacement of the cooled air in the vicinity of the cold body by the warmer air.

4.3.4 Natural Convection



- **Buoyancy forces** are responsible for the fluid motion in natural convection.
- **Viscous forces** appose the fluid motion

Volume expansion coefficient:

A property that represents the *variation of the density of a fluid with temperature at constant pressure:*

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)\bigg|_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)\bigg|_P$$

- This coefficient can be expressed approximately as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad \text{at P} = \text{constant}$$

$$\Rightarrow \boldsymbol{\rho_\infty - \rho = \rho\beta(T_\infty - T)} \quad \ldots\ldots \textbf{(3.4.1)}$$

- Using ideal gas relation, $P = \rho RT$, we get: $\boldsymbol{\beta = 1/T}$

4.3.4 Natural Convection

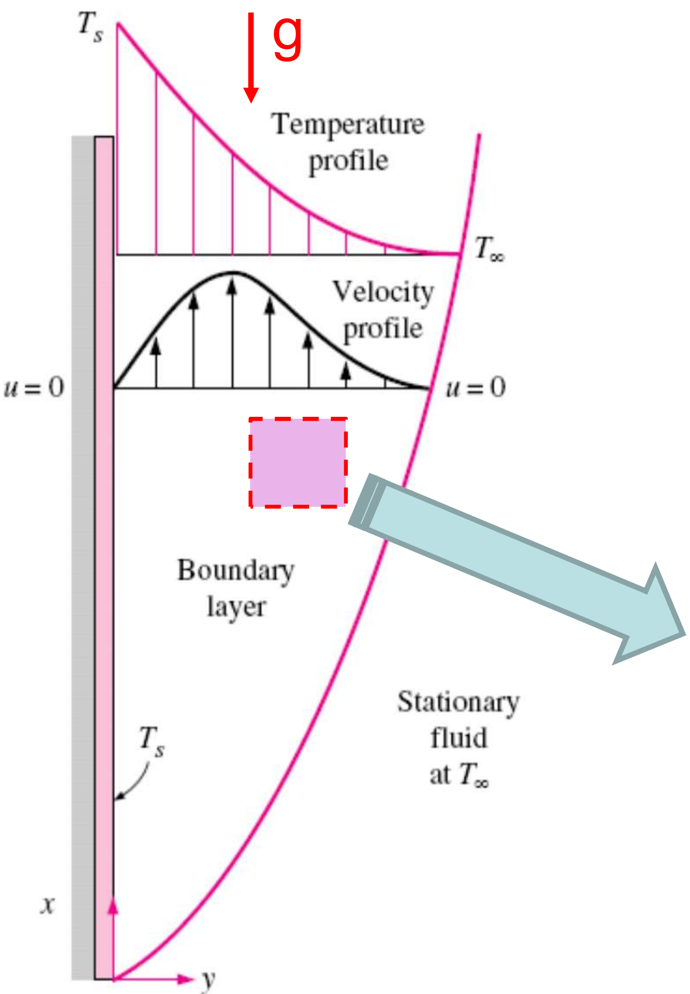


Fig. Vertical hot flat plate immersed in a quiescent fluid body

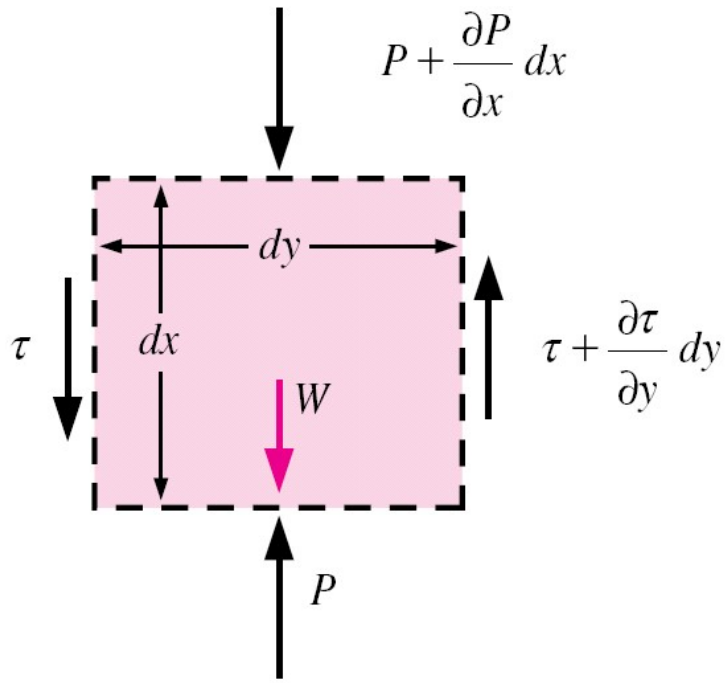
Assumptions:

steady, laminar flow

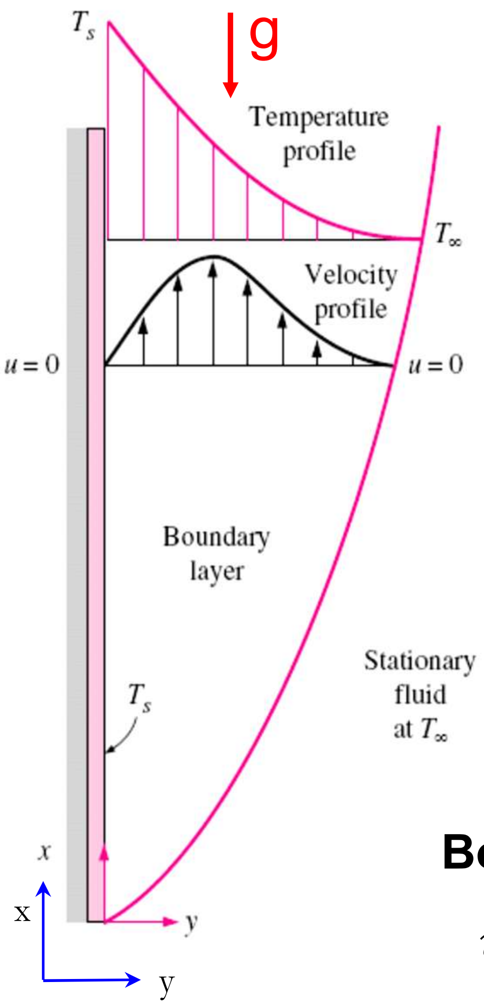
two-dimensional flow

Newtonian fluid, and

constant properties



4.3.4 Natural Convection



The complete set of conservation equations that govern natural convection flow over vertical isothermal plate:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \quad \ldots\ldots (3.4.4)$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \nu\frac{\partial^2 T}{\partial y^2} \quad \ldots\ldots (3.4.5)$$

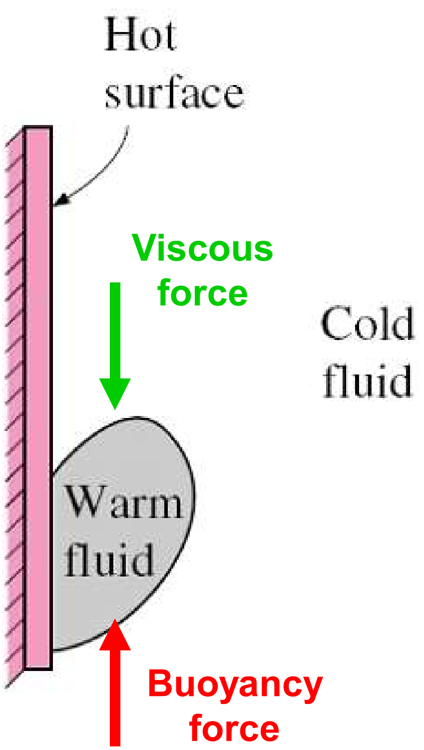
Boundary Conditions:

$u(x, o) = 0,\quad v(x, 0) = 0,\quad T(x, 0) = T_s \quad$ At y = 0

$u(x, \infty) \to 0,\quad v(x, \infty) \to 0,\quad T(x, \infty) \to T_\infty \quad$ At $y \to \infty$

4.3.4 Natural Convection

❑ Grashof Number:



$$\text{Grashof Number} = \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

- g gravitational acceleration, m/s^2
- β coefficient of volume expansion, 1/K
- (β = 1/T for ideal gases)
- T_s temperature of the surface, °C
- T_∞ temperature of the fluid sufficiently far from the surface, °C
- L_c characteristic length of the geometry, m
- ν kinematic viscosity of the fluid, m^2/s

- The flow regime in natural convection is governed by the *Grashof number*
- $Gr_L > 10^9$ flow is turbulent

4.3.4 Natural Convection

- Natural convection heat transfer on a surface depends on:
  - (i) geometry,
  - (ii) orientation,
  - (iii) variation of temperature on the surface, and
  - (iv) thermophysical properties of the fluid.

- The simple empirical correlations for the average *Nusselt number* in natural convection are of the form

$$Nu = \frac{hL_c}{k} = C \cdot \left(Gr_L \cdot \Pr\right)^n = C \cdot Ra_L^n \qquad \text{…… (3.4.6)}$$

- Where Ra_L is the **Rayleigh number** $Ra_L = Gr_L \cdot \Pr = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2}\Pr$

- The values of the constants C and n depend on the geometry of the surface and the flow regime
- The value of n is usually 1/4 for laminar flow and 1/3 for turbulent flow. The value of the constant C is normally less than 1.

4.3.4 Natural Convection

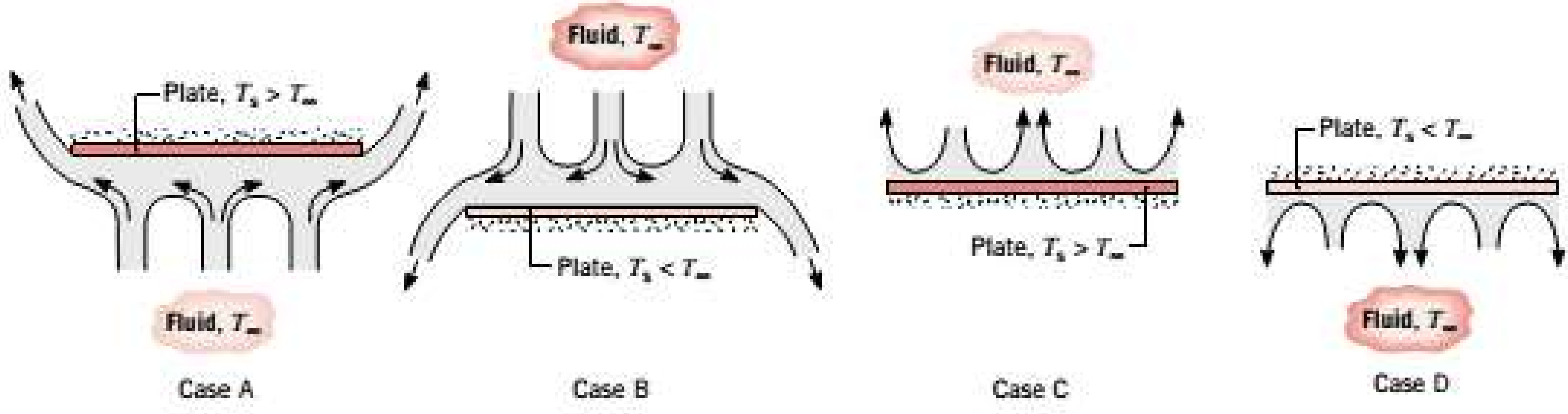


Figure: Free convection buoyancy-driven flows for hot ($T_s > T_\infty$) and cold ($T_s < T_\infty$) horizontal plates:

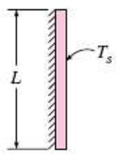
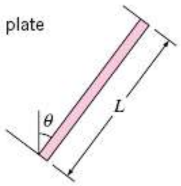
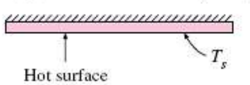
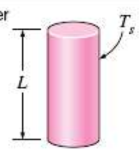
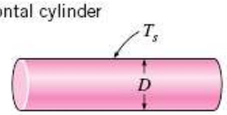
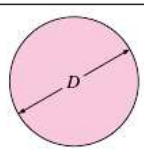
Case A — hot surface facing downwards,
Case B — cold surface facing upwards,
Case C — hot surface facing upwards, and
Case D — cold surface facing downwards.

Reference: Moran et al. (2003) Introduction to Thermal systems Engineering, © John Wiley and Sons Inc. p.443

4.3.4 Natural Convection

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

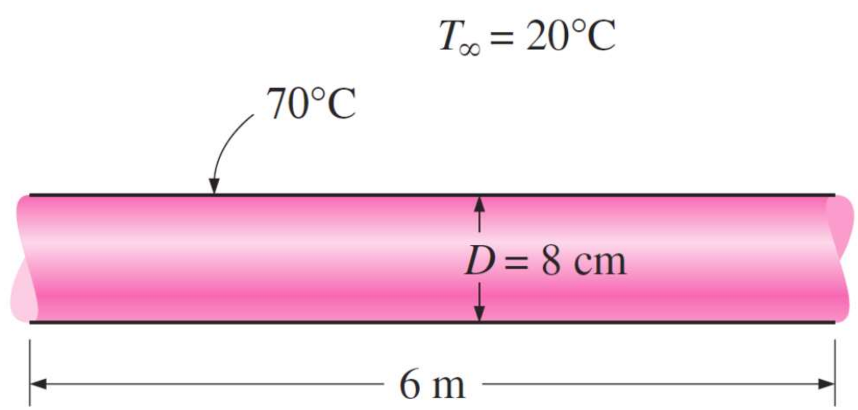
Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{10}–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

All fluid properties are to be evaluated at the film temperature $T_f = (T_s+T_\infty)/2$.

The relations for the constant surface temperature and constant surface heat flux cases are nearly identical.

EP# 3.11 Cengel et al. Example: 9-1

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.



Assumptions:

1. Steady operating conditions exist.
2. Air is an ideal gas
3. The local atmospheric pressure is 1 atm.

Properties

The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$k = 0.02699$ W/m°C   Pr = 0.7241

$\nu = 1.749 \times 10^{-5}$ m^2/s $\beta = 1/T_f = 1/318$

$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\ \mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\mathrm{Pr}$$