Previous Class

Summary of Heat Transfer Processes (Modes of HT)

Mode	Mechanism	Equation	Transport Property or coefficient
Conduction	Energy transfer due to molecular/atomic activity	$q = -k\frac{dT}{dx}$ Fourier's Law	k (W/m.K)
Convection	Energy transfer due to molecular motion and bulk fluid motion	$q = h\Delta T$ Newton's Law	h ($W/m^2.K$)
Radiation	Energy transfer due to exchange of electromagnetic waves (photons)	$q = \sigma\varepsilon T^4$ Stefan-Boltzmann Law	ε (-)

q : Heat flux—rate of heat transfer per unit area, (W/m^2)

ME265: Thermal Engineering & Heat Transfer

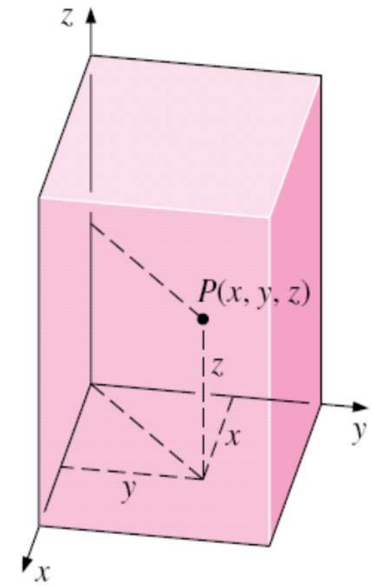
Chapters
1. Energy Scenario
2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

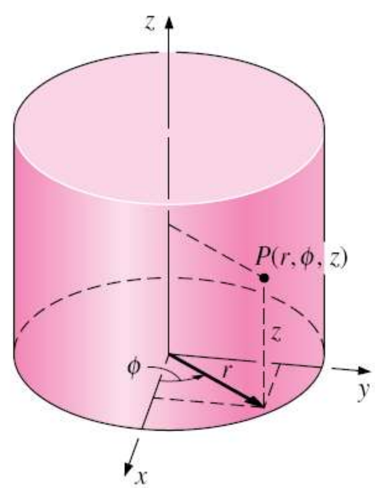
4.1 Introduction	
4.2 Conduction	4.2.1 Conduction Equations 4.2.2 Boundary & Initial conditions 4.2.3 Steady Heat Conduction 4.2.4 Transient Heat Conduction
4.3 Convection	
4.4 Radiation	
4.5 Heat Exchanger	

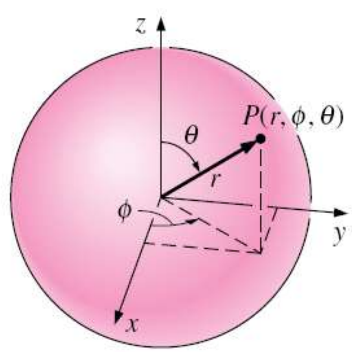
4.2 Conduction Heat Transfer

4.2.1 Conduction Equation / Heat diffusion equation

- **Differential Equation** based on:
  : First Law of Thermodynamics, and
  : Fourier's Law of Conduction
- It is very useful in determining temperature distribution within a solid/stationary liquid
- Solids of different geometries can be analysed using—
  - Cartesian C/S for cubical bodies
  - Cylindrical C/S for cylindrical bodies
  - Spherical C/S for spherical bodies





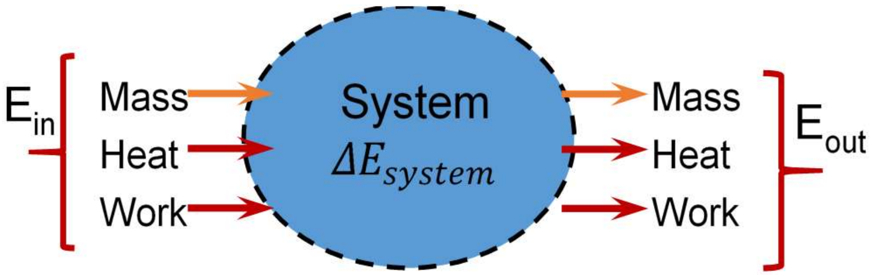


4.2.1 Conduction Equation

❑ Energy Balance

First Law of Thermodynamics:

$$E_{in} - E_{out} = \Delta E_{system}$$



Energy of the system, E_{System}, has two components:

- Stored Energy, E_{st}, in the form of thermal and mechanical
- Energy generation, E_{gen}, in the forms of chemical, nuclear or electrical; *this can be either positive or negative*

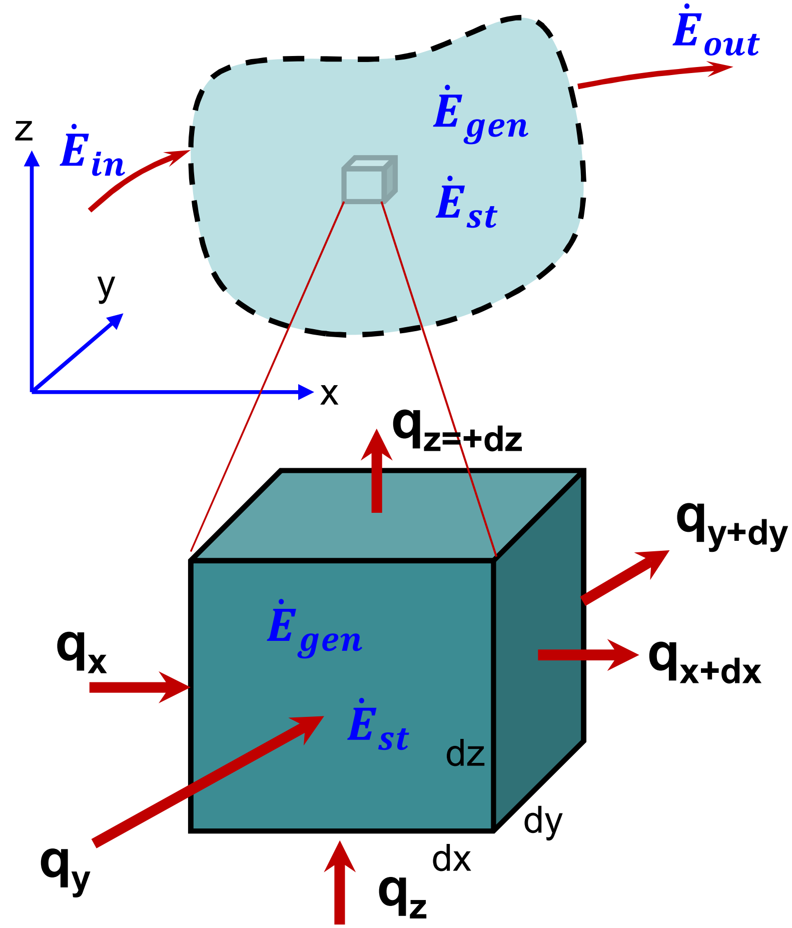
Thus the Energy Balance can be rewritten as:

$$E_{in} - E_{out} + E_{gen} = \Delta E_{st}$$

In the Rate Form:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \qquad \text{.... .... (2.1)}$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad \text{.... ....} \ (2.1)$$



q : Heat fluxes—
⇒ rate of heat transfer per unit area

$$\dot{E}_{in} - \dot{E}_{out}$$
$$= (q_x - q_{x+dx})\mathrm{dy\ dz}$$
$$+ (q_y - q_{y+dy})\mathrm{dz\ dx}$$
$$+ (q_z - q_{z+dz})\mathrm{dx\ dy} \quad \text{.... ....} \ (2.2a)$$

$$\dot{E}_{gen} = \dot{e}_{gen} dx\, dy\, dz \quad \text{.... ....} \ (2.2b)$$

$$\dot{E}_{st} = m\, c \frac{\partial T}{\partial t} = (\rho\, dx\, dy\, dz) c \frac{\partial T}{\partial t} \quad \text{.... ....} \ (2.2c)$$

⇒ **For homogenous solid/medium, ρ and c are constant.**

4.2.1 Conduction Equation

$$\dot{E}_{in} - \dot{E}_{out} = (q_x - q_{x+dx})\,\mathrm{dy\,dz} + (q_y - q_{y+dy})\,\mathrm{dz\,dx} + (q_z - q_{z+dz})\,\mathrm{dx\,dy} \quad \text{.... ....} \ \mathbf{(2.2a)}$$

q : Heat fluxes—rate of heat transfer per unit area

$$\Rightarrow q_x = -k_x \frac{dT}{dx}$$
$$\Rightarrow q_y = -k_y \frac{dT}{dy}$$
$$\Rightarrow q_z = -k_z \frac{dT}{dz}$$

} Fourier's law of HC

Applying Taylor's series expansion and neglected higher order terms:

$$\dot{E}_{in} - \dot{E}_{out} = -\frac{\partial q_x}{\partial x} dx\,\mathrm{dy\,dz} - \frac{\partial q_y}{\partial y} \mathrm{dx\,dy\,dz} - \frac{\partial q_z}{\partial z} \mathrm{dx\,dy\,dz}$$

Introducing Fourier Law, we get:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dx\,\mathrm{dy\,dz} + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) \mathrm{dx\,dy\,dz} + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) \mathrm{dx\,dy\,dz}$$

.... **(2.2d)**

4.2.1 Conduction Equation

From Eq.(2.1), one can get:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + \dot{e}_{gen} = \rho c \frac{\partial T}{\partial t} \qquad \text{.... .... (2.3)}$$

This is general heat conduction equation in Cartesian (Rectangular) Coordinate System

For isotropic solid/medium,

$$k_x = k_y = k_z = k \text{ (constant)}$$

Therefore, for homogeneous, isotropic solid/medium, Eq. (2.3) can be simplified to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{.... .... (2.4)}$$

Heat Generation

- Heat generation is a *volumetric phenomenon* (W/m^3 or Btu/h ft^3).
- The *total* rate of heat generation in a medium of volume *V* can be determined from

$$\dot{E}_{gen} = \int_V \dot{e}_{gen} dV$$

- In the special case of *uniform* heat generation $\dot{E}_{\text{gen}} = \dot{e}_{gen}V$

Examples:

- **Resistance wires** where electrical energy being converted to heat at a rate of *I^2R (Joule heating)*
- **Fuel elements** of nuclear reactors where nuclear fission serves as the *heat source*
- **Exothermic chemical** reactions where chemical reaction serves as a *heat source* for the medium.

Heat Generation Examples:

4.2.1 Conduction Equation

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad \text{.... .... (2.1)}$$

Fourier's law of HC

$$q_x = -k_x \frac{\partial T}{\partial x}$$
$$q_y = -k_y \frac{\partial T}{\partial y}$$
$$q_z = -k_z \frac{\partial T}{\partial z}$$

For isotropic solid,

$$k_x = k_y = k_z = k$$

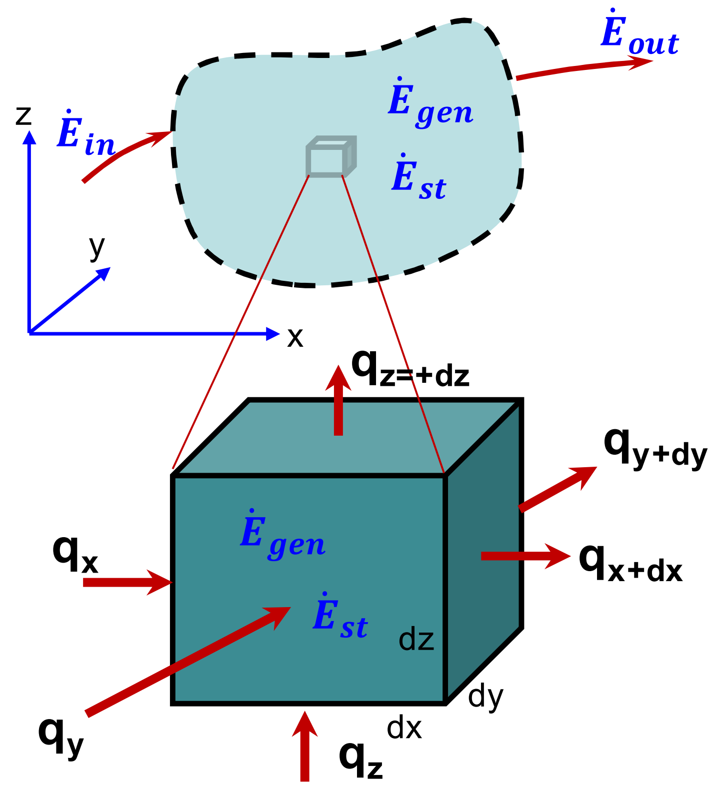


Fig. 1: Cartesian C/S

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{.... .... (2.4)}$$

$\dot{e}_{gen}$ → Volumetric heat generation, W/m^3
k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s

4.2.1 Conduction Equation

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad \text{.... (2.1)}$$

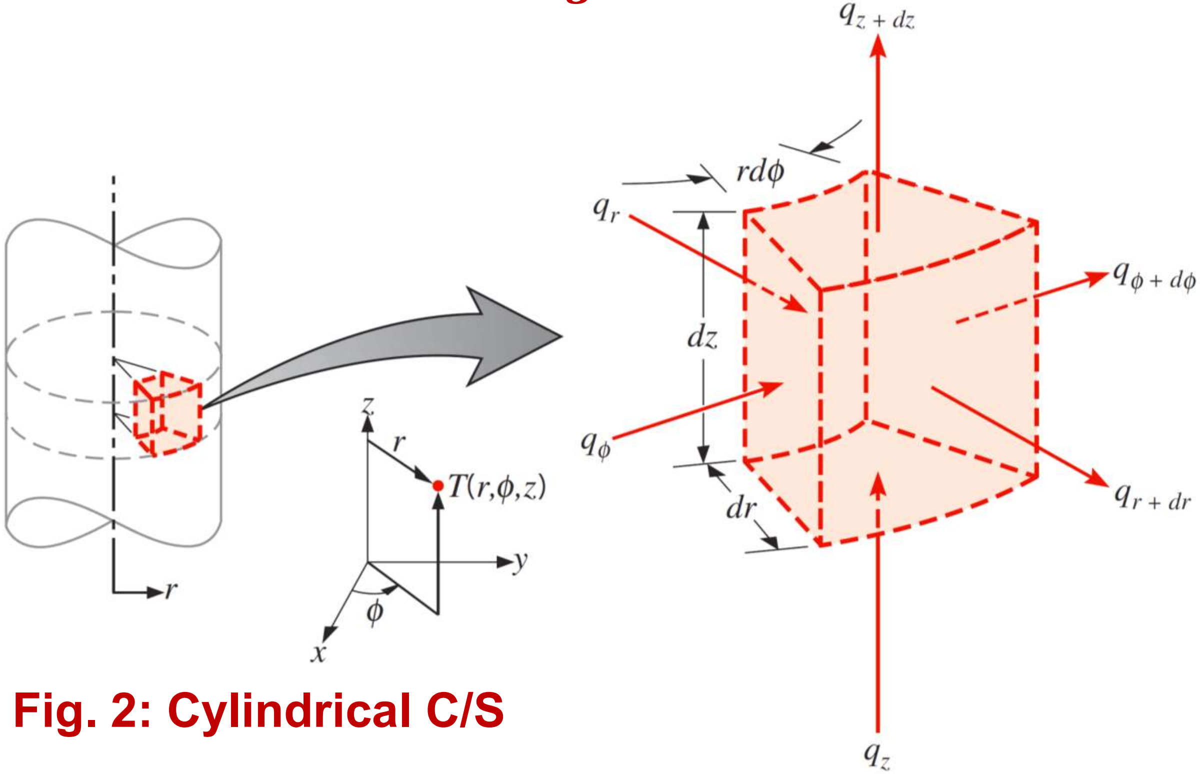


Fig. 2: Cylindrical C/S

Fourier's law of HC

$$q_r = -k_r \frac{\partial T}{\partial x}$$

$$q_\emptyset = -k_\phi \frac{\partial T}{r\partial \emptyset}$$

$$q_z = -k_z \frac{\partial T}{\partial z}$$

For isotropic solid,
$k_r = k_\emptyset = k_z = k$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \emptyset^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{.... .... (2.5)}$$

$\dot{e}_{gen}$ → Volumetric heat generation, W/m^3
k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s

4.2.1 Conduction Equation

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad \text{..... (2.1)}$$

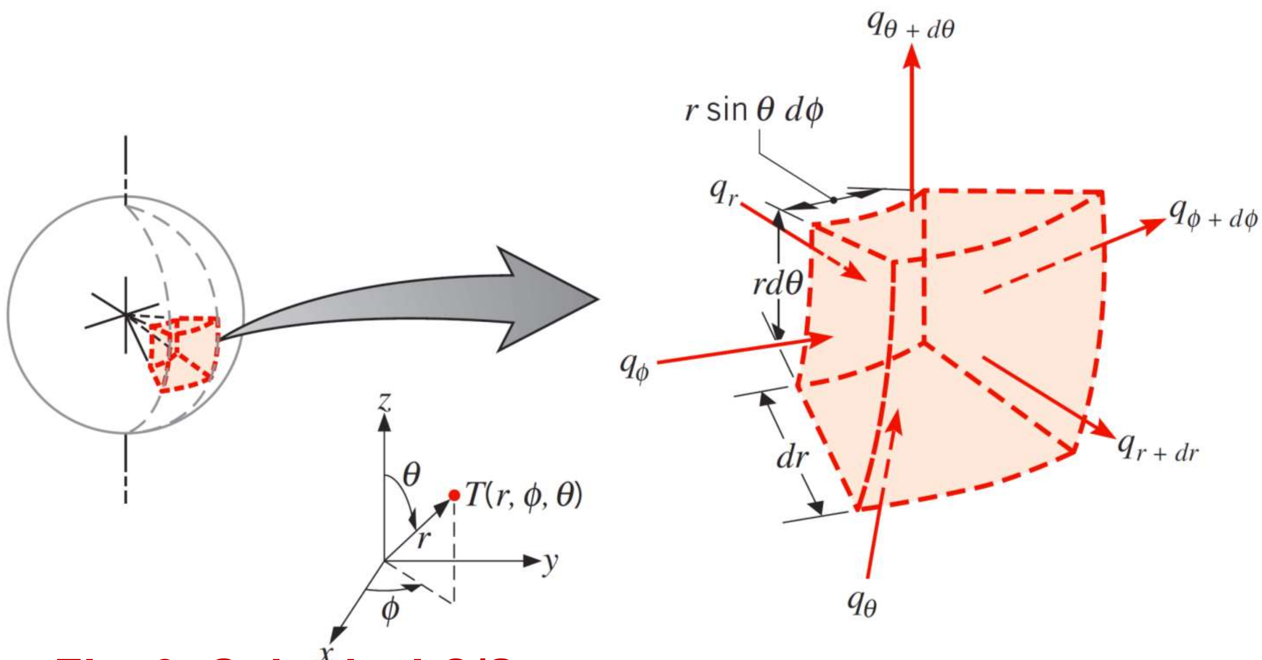


Fig. 3: Spherical C/S

Fourier's law of HC

$$q_r = -k_r \frac{\partial T}{\partial r}$$

$$q_\phi = -k_\phi \frac{\partial T}{r \sin\theta \, \partial\phi}$$

$$q_\theta = -k_\theta \frac{\partial T}{r \, \partial\theta}$$

For isotropic solid,
$k_r = k_\emptyset = k_\theta = k$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \emptyset^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{.... .... (2.6)}$$

$\dot{e}_{gen}$ → Volumetric heat generation, W/m^3
k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s