

# Impulse and Momentum Principles

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**Turbo-machines** are devices in which energy is transferred either to, or from, a continuously flowing fluid by the dynamic action of moving blades on the runner. The word turbo or turbinis is of Latin origin and implies that spins or whirls around.

The turbomachines may be *classified* as :

**(a) Open and enclosed machines :**

**(i) open turbomachines** are those which influence an indefinite quantity of fluid .e.g., propellers, windmills and unshrouded fans. Such machines come generally under the category of aerodynamics.

**(ii) Enclosed turbomachines** in which a finite quantity of fluid passes through a casing in unit time.

**(b) Absorption and production of power :**

**(i)** Those *which absorb power* to increase the fluid pressure or head e.g., pump, ducted fans and compressors.

**(ii)** Those *which produce power* by expanding fluid to a lower pressure or head e.g., hydraulic, steam and gas turbines.



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### (c) Type of fluid handled :

- (i) Those which *handle water* e.g., pumps and hydraulic turbines.
- (ii) Those which *handle steam* e.g., steam turbines.
- (iii) Those *handle air or gas* e.g., ducted fans, compressors and gas turbines.

**Hydraulic Machines**--hydraulic machine is a general term used for all *devices/machines handling liquids*.

### Dynamic Action of Fluid

A stream of fluid entering in a machine such as a hydraulic or steam turbine, a pump or fan, has more or less a definite direction. **A force is always required to act upon the fluid to change its velocity either in direction or in magnitude.** According to Newton's third law, an equal and opposite force is exerted by the fluid upon the body that causes the change. This force exerted by virtue of fluid motion is called a *Dynamic force* and must be distinguished from hydrostatic pressure. Whereas *hydraulic pressure implies no motion, dynamic force always involves a change in velocity and thus a change in momentum.*

The power is determined from the dynamic force or forces which are being exerted by the flowing fluid on the boundaries of flow passage and which are due to the change of momentum. These are determined by applying “*Newton's Second Law of Motion.*”

Momentum may be linear or angular. In fact **angular momentum is moment of linear momentum**. Rate of change of linear momentum is equal to the force which is responsible for this change; while rate of change of angular momentum will be equal to the torque of a fluid mass.

## Newton's Second Law of Motion, Linear Momentum Equation and Impulse-Momentum Equation:

The fundamental principle of dynamics is *Newton's Second Law of Motion* which states that “the rate of change of momentum is proportional to the applied force and takes place in the direction of the force” i.e. the resultant external force  $F_x$  acting on the particles of mass  $m$  along any arbitrarily chosen direction  $x$  is equal to the time rate of change of linear momentum of the particle in the same direction i.e.  $x$  direction.

Momentum of a body is the product of its mass and velocity. Let  $m$  be the **mass** of fluid moving with **velocity  $v$**  and let the **change of velocity be  $dv$**  in **time  $dt$** .

So change of momentum =  $m \cdot dv$

*and* rate of change of momentum =  $m \cdot (dv/dt)$

According to the above law

*Dynamic force applied in x-direction = Rate of change of momentum in x-direction*

$$F_x = m \cdot \frac{dv_x}{dt} \quad \text{----- (1)}$$

where the suffix  $x$  denotes components in  $x$  direction. This equation is known as **Linear momentum equation** and can also be written as

$$F_x \cdot dt = m \cdot dv_x \quad \text{----- (2)}$$

**Equation (2)** is known as **Impulse momentum Equation** which states

*“Impulse of dynamic force = resulting change in momentum of body.”*

The left hand term of this equation is the *product of force and the time increment during which it acts*. This is known as the impulse of applied force. The right hand term is the resulting change in momentum.

Since velocity is a vector quantity, so consequently any change in magnitude or in direction or in both will change the velocity, hence momentum.

## Limitation of Application of Newton's second Law of motion

Newton's second law of motion (refer Equ. 1) is generally applicable to a control mass system (of constant mass). This may be written as

$$F_x = m \cdot \frac{dv_x}{dt}$$

where  $m$  is the constant mass of the system,  $\sum F_x$  is the algebraic sum (or resultant) of all surface forces as well as body forces such as gravity acting on mass  $m$  in an arbitrary direction  $x$ .  $v_x$  is the velocity of the centre of mass of the system in  $x$ -direction.  $dv_x$  is the change in  $v_x$  in time  $dt$ .

**Control volume** is a specific region in space, its size and shape being *entirely arbitrary*, however these are made to coincide with solid boundaries. The boundary of a control volume is its control surface. For a control volume with fluid entering with uniform velocity  $v_{x1}$  and leaving after time  $t$  with uniform velocity  $v_{x2}$ , Equ (1) gives

$$\Sigma F_x = \frac{m}{t} (v_{x2} - v_{x1}) \quad \text{----- (3)}$$

Since the dimensions of  *$m/t$  is mass per unit time that is mass flow  $\rho Q$* , the algebraic sum of external forces on the fluid in steady flow is equal to the mass flow multiplied by the velocity change in  $x$ -direction, i.e.

$$\Sigma F_x = \rho Q (v_{x2} - v_{x1}) \quad \text{----- (4)}$$

where  $Q$  is the rate of flow and  $\rho$  the density.

**This is one dimensional form of steady flow momentum equation**, which states that the *algebraic sum of the body forces on the matter within the control volume, plus the forces acting on the control surface, both in the  $x$ -direction is equal to the net out-going flux of momentum in  $x$ -direction*. This equation is important in the study of turbomachines as it enables to determine the forces developed by the flow in a fluid machine.

$$\Sigma F = \frac{d(mV)_{out}}{dt} - \frac{d(mV)_{in}}{dt} \quad \text{----- (4a)}$$

This can also be written as  $\Sigma F = \rho_2 Q_2 V_2 - \rho_1 Q_1 V_1$

If the fluid is incompressible, then  $\Sigma F = \rho Q (\Delta V)$

In this case  $\Delta V$  should be taken as the **vectorial addition of  $V_1$  and  $V_2$**  and **the force will be in the direction of the resultant** of  $V_1$  and  $V_2$ .

In case the forces in the Cartesian co-ordinate directions is required, the equation in scalar form is written as

$$\Sigma F_x = \rho Q \Delta u$$

$$\Sigma F_y = \rho Q \Delta v$$

$$\Sigma F_z = \rho Q \Delta w$$

where  $u$ ,  $v$  and  $w$  are the components of velocity in the  $x$ ,  $y$  and  $z$  directions.

**External forces  $F_x$** , may be of three kinds :

*pressure forces*, *inertia forces (body forces)* and *drag forces*.

- (a) **Pressure forces** are those acting between the fluid and boundary surfaces or between any two adjacent fluid layers.
- (b) **Inertia forces** are those caused by the action of gravity and/or centrifugal effects. These are also known as **body forces**.
- (c) **Drag forces** are those existing between boundary surfaces and flow. For example forces acting on model hanging in a wind tunnel with the outside support. These are also known as **viscous forces**.

## Dynamie Force Exerted by Fluid Jet on Plate :

### (a) Plate Normal to Jet

A fluid jet issues from a nozzle and strikes a flat plate with a velocity  $v$ . *The plate is held stationary and perpendicular to the centre line of the jet.*

Let  $Q$  = quantity of fluid falling on the plate, in  $\text{m}^3/\text{sec}$  = volume / time

$\gamma$  = specific weight of fluid in  $\text{kg}/\text{m}^3$  ;

then  $\gamma.Q$  = weight of fluid in  $\text{kg}/\text{sec}$  ;

And  $\gamma.Q/g = \rho Q$  = mass of fluid per sec.

Velocity of fluid in  $x$ -direction before striking the plate =  $v$  m/sec

Final velocity of fluid in  $x$ -direction after striking the plate = 0 m/sec

**Dynamic force on the fluid by the plate** = Change of momentum/sec  
 = mass striking the plate/sec  
 × change of velocity normal to the plate

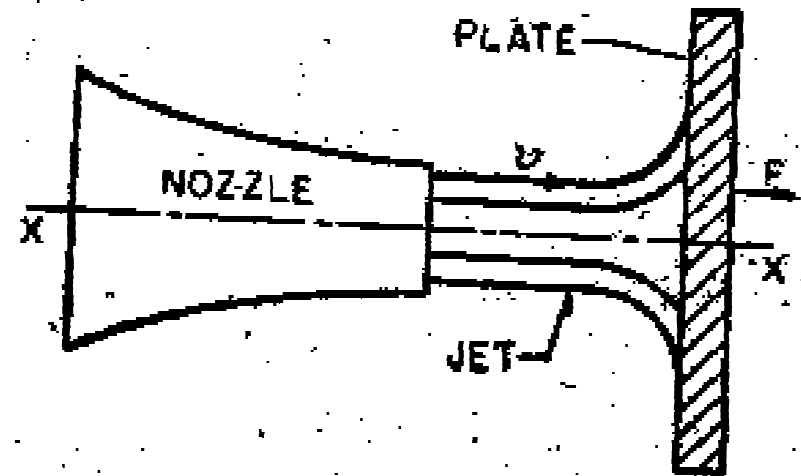
$$\Sigma F_x = \rho Q (v_{x_2} - v_{x_1})$$

The change of velocity is the difference between its final and initial values. Then

$$-F_x = \rho Q (0 - v) = -\rho Q \cdot v$$

The minus sign on right hand side of the equation indicates that the velocity is decreasing, while this sign used with  $F_x$  indicates that the force is acting in the negative direction of  $x$ -axis. The force exerted on the fluid, by the plate is thus

$$-F_x = -\rho Q \cdot v$$



Here the plate is responsible for changing the velocity of jet, therefore the force is exerted by the plate on the fluid. Since the final velocity is less than the initial velocity, the force exerted on the fluid by the plate is a retarding force, thus it acts in the opposite direction to that of flow.

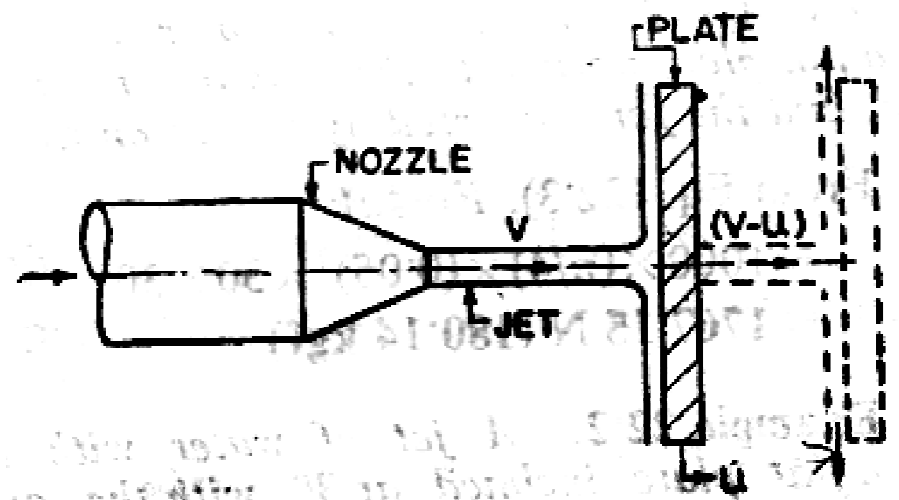
Now **the force exerted by the fluid on the plate is given by** “Newton’s Law of Action and Reaction” which will be equal and opposite, namely:

$$F_{\text{on}} = \rho Q \cdot v = \rho A v^2 \quad \text{where } A \text{ is the x-section area of jet}$$

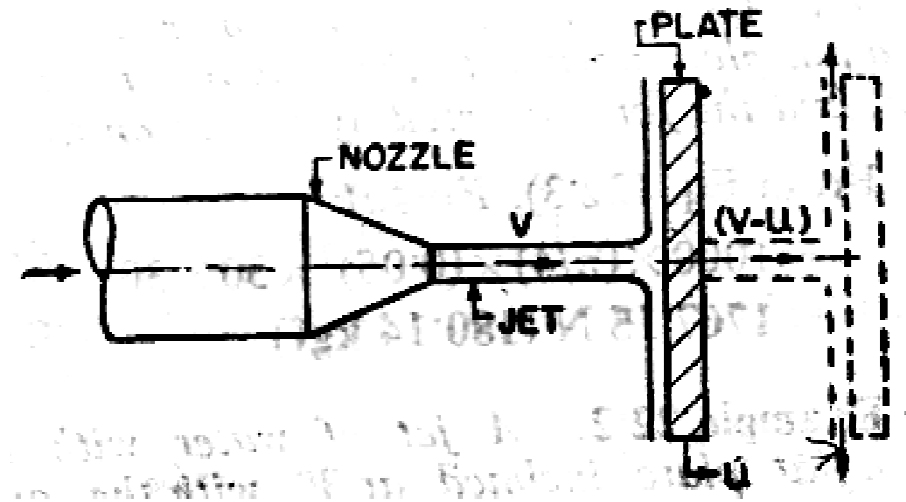
*Half of the quantity of water  $Q$  falling on the stationary vertical plate will move upward and remaining half downward, along the surface of the plate. Since the plate is stationary, work done on the plate is zero.*

**(b) Thrust on Moving Flat Plate Held Normal to the Direction of Jet**

Consider the case when a flat plate held normal to the jet moves with a velocity  $u$  in the direction of the jet.



The effective velocity with which the jet strikes the plate is the relative velocity  $(V-u)$ . The plate moves away from the jet before it strikes it.



The mass of fluid striking the plate per second,  $M = \rho A(V-u)$

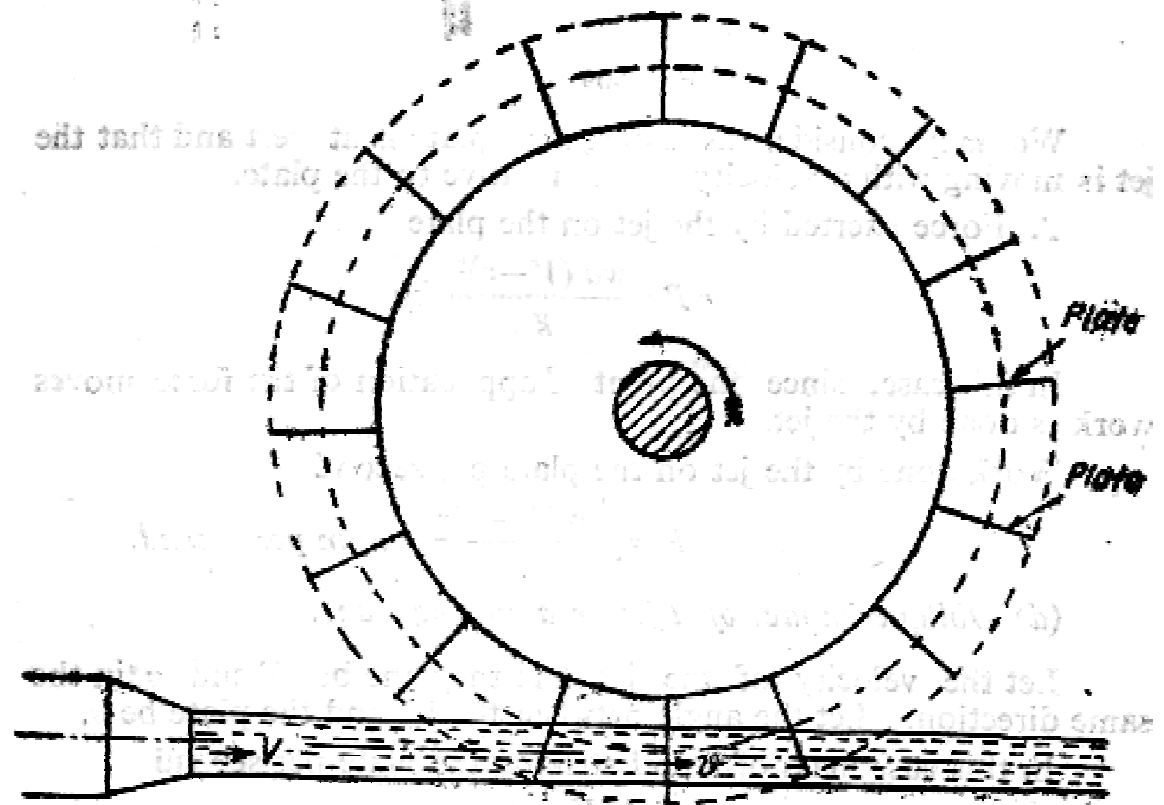
Thrust exerted on the plate in the direction of jet is

$$F = \rho A(V-u)[(V-u) - 0] = \rho A(V-u)^2$$

Work done by the jet per second = Thrust  $\times u = \rho A(V-u)^2 u$

However, it is not practically feasible because the distance between the plate and nozzle is constantly increasing by  $u$  m/sec. If, however, a *series of plates* (next Fig.) were so arranged that (fitted on the wheel) each plate appeared successively before the jet in the same position and always moving with a velocity  $u$  in the direction of jet, then whole flow from the nozzle is utilized by the plates.

In this case a number of flat plates are radially mounted over a wheel. The wheel is supported over a shaft, with a suitable bearing to afford easy rotation of the wheel. The jet moving at a velocity strikes the plates in succession causing the wheel to rotate.



*Velocity of the jet before striking the wheel =  $V$*

*Velocity of the jet after striking the wheel =  $u$*

*= Velocity of the plates at the impact point.*

Thus mass of water per second striking the plate would be  $= \rho AV$

**Thrust on the Plates,  $F$  = Mass striking the wheel per second x**

**Change in velocity,**

$$= \rho AV(V-u)$$

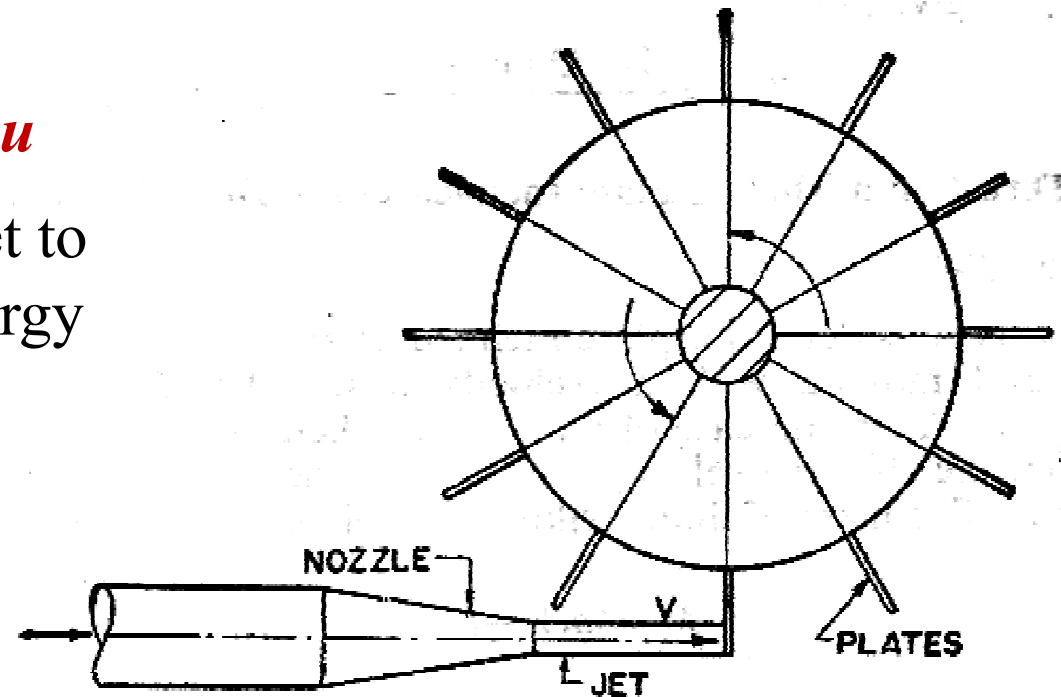
$$\text{Work done per second} = F \times u$$

$$= \rho AV(V-u)u$$

Evidently, energy supplied by the jet to the wheel is equal to the kinetic energy of the jet. Thus **Energy Input**

$$= \frac{1}{2} mV^2 = \frac{1}{2} \times \rho AV \times V^2$$

$$= \frac{1}{2} \rho AV^3$$



**Efficiency of the wheel,**

$$\eta = \frac{\text{Work done}}{\text{Input}}$$

$$\eta = \frac{\rho aV(V-u)u}{\frac{1}{2} \times \rho aV(V^2)}$$

$$\eta = \frac{2u(V-u)}{V^2}$$

For a given jet velocity, **the efficiency will be a maximum if**

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du} \left[ \frac{2u(V-u)}{V^2} \right] = 0$$

$$2V - 4u = 0$$

$$u = V/2$$

**For the maximum efficiency of the wheel, the peripheral speed of the wheel ( $u$ ) is equal to one-half the jet velocity. The maximum efficiency is given by**

$$\eta_{\max} = \frac{2(V/2)(V - V/2)}{V^2} = \frac{1}{2} \text{ or } 50\%$$

The wheel with plates (vanes) provided as in this case and subjected to impact by a jet is called a **water wheel**. When the jet strikes the wheel at the bottom as in this case, the device is called ***an undershot water wheel***. When the jet strikes the wheel at the top, the device is called ***an overshoot water wheel***.

## **Fluid Jet on Curved Plate**

### **(a) Stationary Plate**

The jet impinges on a curved plate (as shown in following fig.) at an angle  $\alpha_1$ , and is deviated to an angle  $\alpha_2$  both angles being measured with respect to X-direction. Let  $v_1$  and  $v_2$  be the velocities of jet at inlet and outlet respectively. The velocity of jet at **inlet  $v_1$  and the velocity of jet at outlet  $v_2$  will be same as long as there is no friction on the plate.**

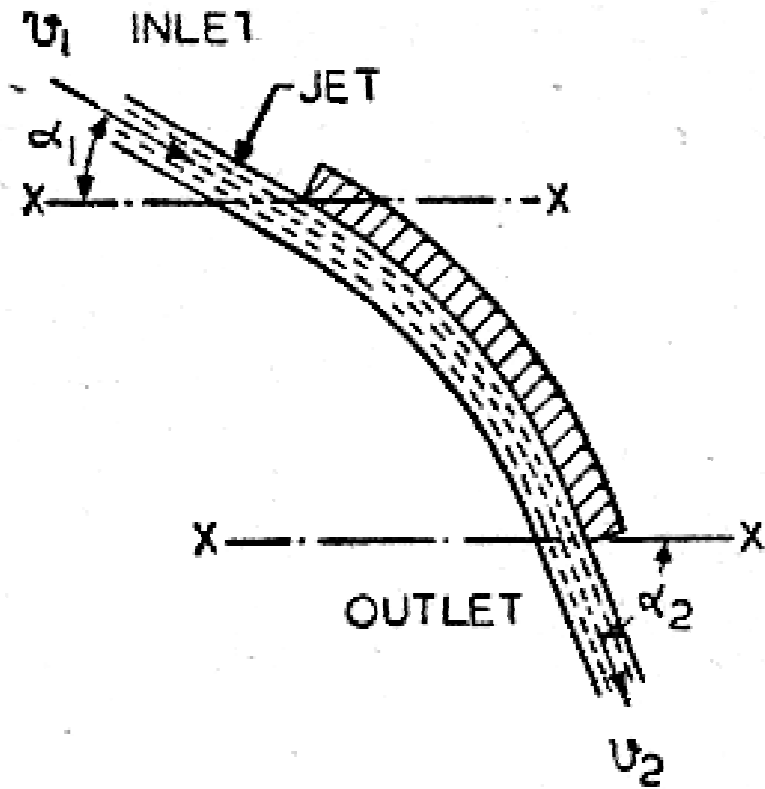


Fig: Jet falling on Stationary Curved Plate with Acute Discharge Angle.

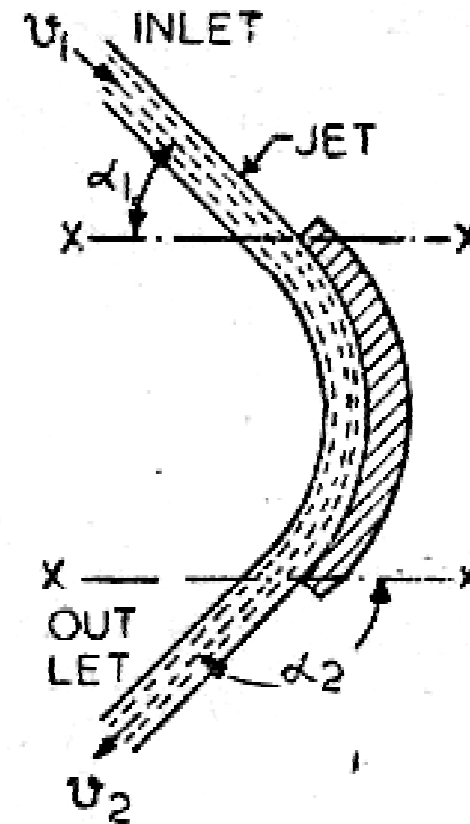


Fig: Jet falling on Stationary Curved Plate with Obtuse Discharge Angle.

Velocity of jet at inlet in X-direction =  $v_1 \cos \alpha_1$

velocity of jet at outlet in X-direction =  $v_2 \cos \alpha_2$

**Force exerted on the plate by the jet in X-direction**

**$F_x = \text{Mass impinging on the plate per second} \times \text{Change in velocity in direction} = \rho Q (v_1 \cos \alpha_1 - v_2 \cos \alpha_2)$**

## Force exerted on the jet by the plate in X-direction

$$F_x = \rho Q (v_2 \cos\alpha_2 - v_1 \cos\alpha_1)$$

where  $Q = av_1$  = quantity of water per second falling on the plate.

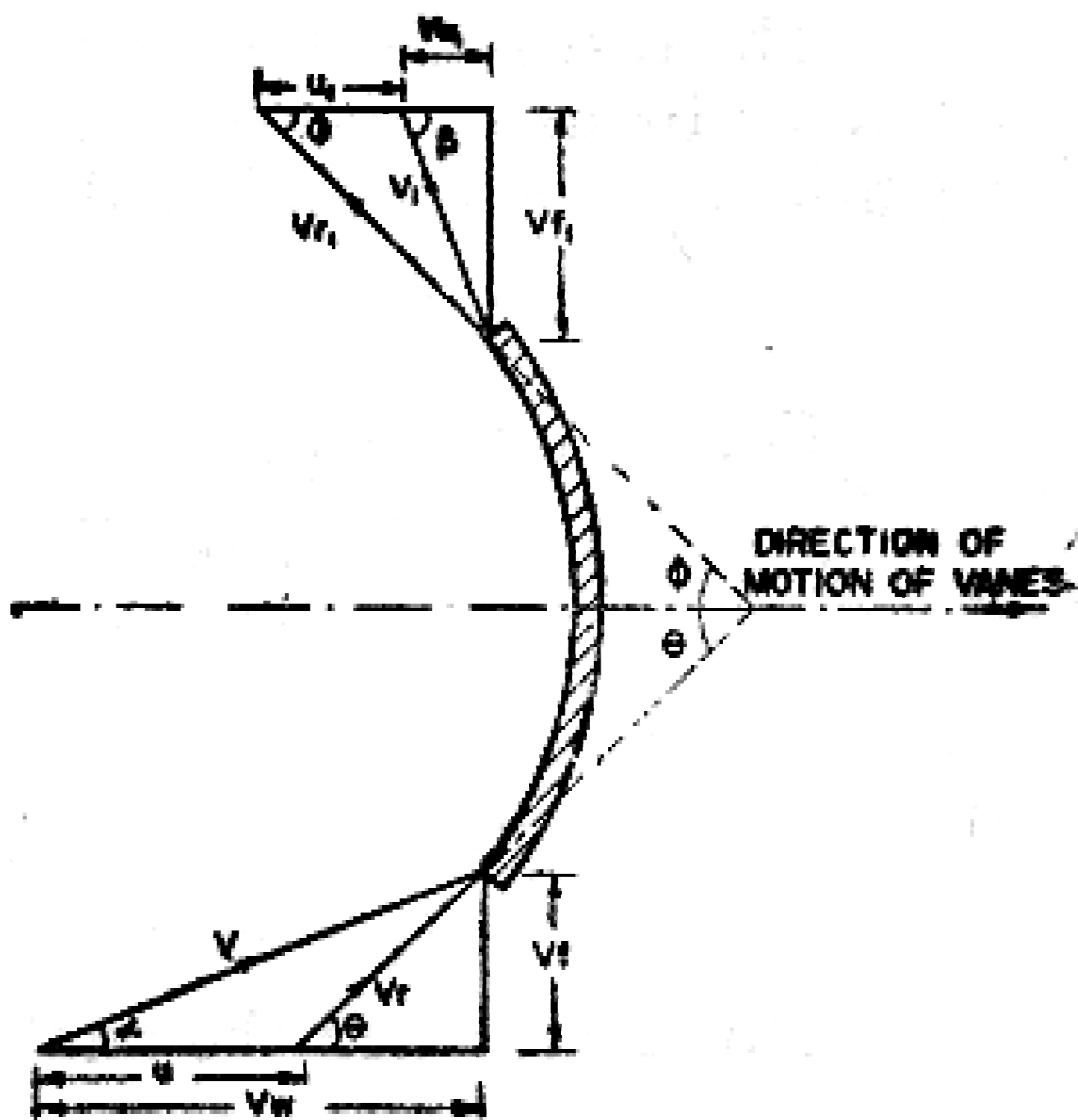
## Force exerted on the plate by the jet in X-direction

$$F_x = \rho av_1 (v_1 \cos\alpha_1 - v_2 \cos\alpha_2)$$

If the curvature of the plate at outlet is such that outlet angle  $\alpha_2$  is more than  $90^\circ$ , then the second term in the bracket i.e.  $v_2 \cos\alpha_2$  will be negative. Hence, in order to get more force, the curvature of the plate should be such that the outlet angle  $\alpha_2$  is obtuse. *Since the plate is stationary, work done on the plate is zero.*

## (b) Jet Striking the Moving Curved Vane

Let us consider the case when the jet strikes the **moving curved vane** at one tip and leaves at other. *The effective velocity with which the jet strikes the vane is the relative velocity ( $V_r$ ).* The curvature of the plate at the point where the jet strikes may or may not make the same angle with x-direction. The jet leaves the vane with the relative velocity  $V_{r1}$ .



The relative velocity  $V_r$ , and the absolute velocity at the exit may be obtained by [drawing the velocity triangle at the inlet and at the outlet.](#) Fig. shows the velocity triangles.

The following symbols are used in the velocity triangles:

$V, V_1$ : *Absolute velocities of the jet* at the inlet and outlet respectively.

$u, u_1$ : *Peripheral velocities of the vanes* at the inlet and outlet respectively.

$V_r, V_{r1}$ : *Relative velocities of the jet at the inlet and outlet respectively.*

*(The relative velocity is the vectorial sum of the absolute velocity and the reverse of the peripheral velocity)*

$V_f, V_{f1}$ : *Velocities of flow* at the inlet and outlet respectively. (The *velocity of flow is the component of the absolute velocity normal to the direction of motion.*)

$V_w, V_{w1}$ : *Velocities of whirl* at the inlet and outlet respectively. (The *velocity of whirl is the component of the absolute velocity in the direction of motion*)

$\theta, \phi$  : Vane *Tip angles* at the inlet and outlet respectively

$\alpha, \beta$  : *Angles which the absolute velocities make* at the inlet and outlet respectively.

**All angles are measured with the direction of motion of vane.**

The **relative velocity approach** used in previous cases may be adopted but usually it is more convenient to use the *velocities of flow and whirl obtained from the velocity triangles*. This approach is known as **velocity triangle approach** and is explained below.

The jet enters the vane **without shock** if the relative velocity  $V_r$  makes an angle  $\theta$  with the direction of motion. The jet glides over the vane and leaves with a velocity  $V_{r1}$ . *If the vane is smooth, the relative velocity remains constant*, i.e.,  $V_r = V_{r1}$ . The jet will leave the vane without shock if the relative velocity  $V_{r1}$  makes an angle  $\phi$  with the direction of motion.

The *thrust on the vane in the direction of motion may be obtained by using the velocity triangles and the impulse momentum equation*.

The mass of fluid striking the vane per second,  $M = \rho a V_r$

$$\begin{aligned} \text{Force on the vane in the direction of motion, } F_x &= M[V_w - (-V_{w1})] \\ &= \rho a V_r [V_w + V_{w1}] \end{aligned} \quad \text{----- (1)}$$

Equation (1) is applicable **when the angle  $\beta < 90^\circ$** .

$$\text{If the angle } \beta > 90^\circ, \quad F_x = \rho a V_r [V_w - V_{w1}] \quad \text{---- (2)}$$

**When the angle  $\beta = 90^\circ$ ,  $F_x = \rho a V_r V_w$  ----- (3)**

**Thus, the general expression for the thrust is,  $F_x = \rho a V_r [V_w \pm V_{w1}]$  ----- (4)**

**Let us consider the case when  $u = u_1$ .**

***Work done per second* =  $F_x \times u = \rho a V_r [V_w \pm V_{w1}]u$  ----- (5)**

**Again, the work done is also equal to the change in the kinetic energy.**

Thus ***work done per second* =  $\frac{1}{2} (\rho a V_r) [V^2 - V_1^2]$  ----- (6)**

From equ (5) and (6):  $[V_w \pm V_{w1}]u = \frac{1}{2}[V^2 - V_1^2]$   
 $2[V_w \pm V_{w1}]u = V^2 - V_1^2$

Now **Input Energy = Kinetic energy of jet at inlet =  $\frac{1}{2} (\rho a V_r) V^2$**

**Efficiency of the vane,  $\eta = \text{work done per second} / \text{Input Energy}$**

$$\begin{aligned} \eta &= \rho a V_r [V_w \pm V_{w1}]u / \frac{1}{2}(\rho a V_r) V^2 \\ &= 2[V_w \pm V_{w1}]u / V^2 \end{aligned} \quad \text{----- (7)}$$

Also  $\eta = [V^2 - V_1^2] / V^2$  ----- (8)

**Note that the above expressions are applicable only if  $V_r = V_{r1}$  i.e. *the no friction.***

### (c) Series of Vane

If a series of vanes is fixed radially to the rim of a wheel, there is always one vane after another facing the jet as shown in following Fig. Thus the entire fluid is utilized. This arrangement of vanes is used in radial flow turbines. *Depending upon whether the jet enters the outer periphery or the inner periphery, the turbine is termed the inward flow or outward flow*. The fig. shows an inward flow impulse turbine.

Let  $R$  and  $R_1$  be the radii of wheel at the inlet and outlet respectively. Let  $\omega$  be the *angular velocity of the wheel*. Thus

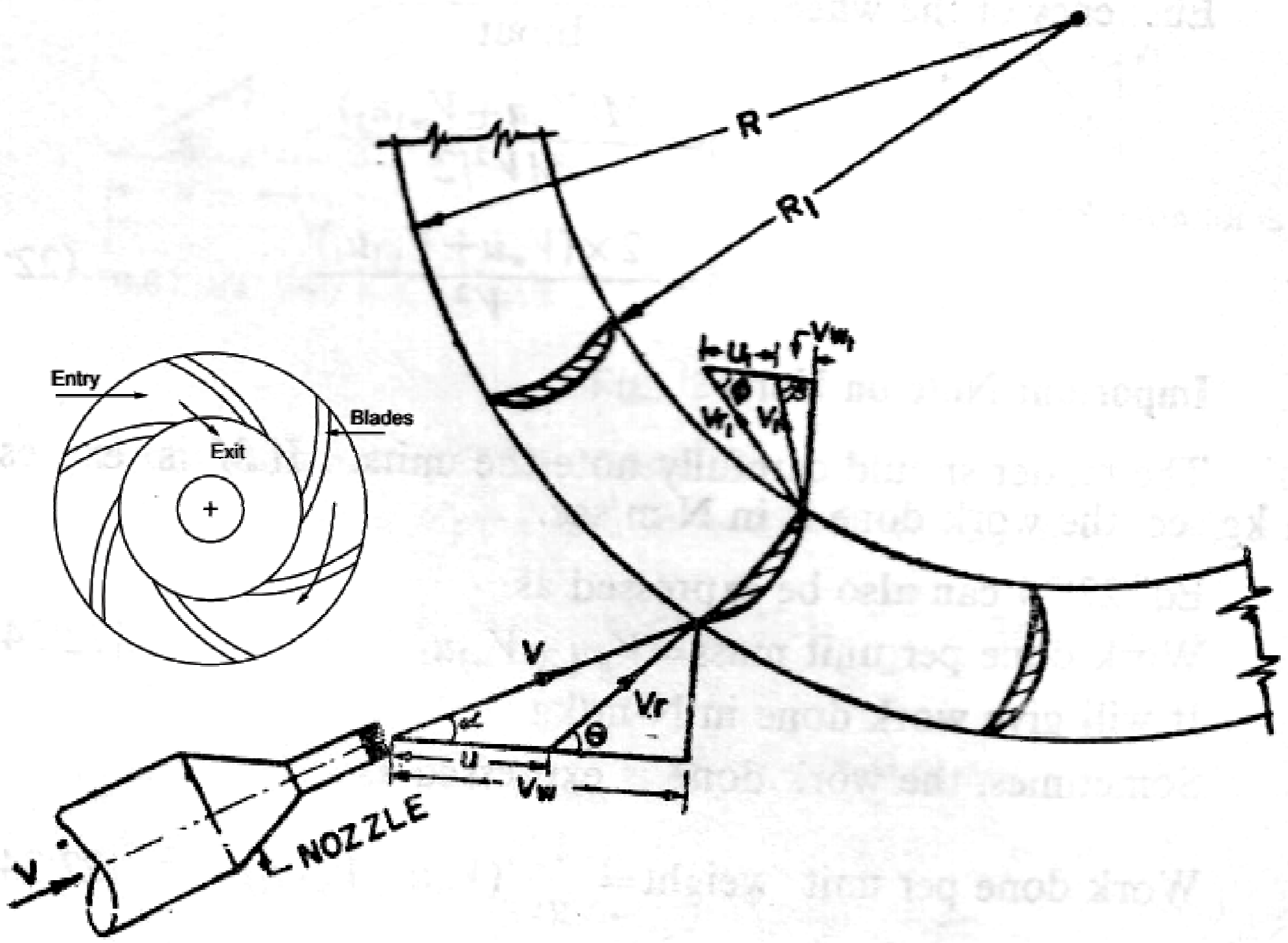
$$u = \omega R = \frac{2\pi N}{60} R$$

----- (a)

$$u_1 = \omega R_1 = \frac{2\pi N}{60} R_1$$

----- (b)

Where  $N$  is *the speed of the wheel* in r.p.m. Let  $M$  be the mass of fluid striking per second. The momentum in the tangential direction at the inlet is  $MV_w$ , and the *moment of momentum about the centre* is  $M(V_w R)$ . The moment of momentum at the outlet ( $-MV_{w1} R_1$ ).



Torque on the wheel = Change of moment of momentum  
or

$$\begin{aligned} T &= MV_w R - (-MV_{w1} R_1) \\ &= M(V_w R + V_{w1} R_1) \end{aligned}$$

where  $M$  is mass of fluid striking per second.

Work done on the wheel per second

$$\begin{aligned} &= \text{Torque} \times \text{Angular Velocity} \\ &= M(V_w R + V_{w1} R_1) \omega \end{aligned}$$

From Esq. (a) and (b),

$$u = \omega R \text{ and } u_1 = \omega R_1.$$

Therefore, work done per second =  $M(V_w u + V_{w1} u_1)$ .

The above equation has been developed on the assumption that the angle  $\beta$  is less than  $90^\circ$ . If the angle  $\beta$  is greater than  $90^\circ$ ,

$$\text{Work done per second} = M(V_w u - V_{w1} u_1)$$

The general expression for the work done on the wheel is given by

$$\text{Work done per second} = M(V_w u \pm V_{w1} u_1) \dots (22.34)$$

This is the well-known Euler momentum equation for turbines.

$$\text{Efficiency of the wheel } (\eta) = \frac{\text{Work done}}{\text{Input}}$$

$$\eta = \frac{M(V_w u \pm V_{w1} u_1)}{MV^2/2}$$

$$\eta = \frac{2 \times (V_w u \pm V_{w1} u_1)}{V^2} \quad \dots(22.35)$$

### Important Note on Units

The reader should carefully note the units. If  $M$  is expressed in kg/sec, the work done is in N-m/sec.

Eq. 22.34 can also be expressed as

$$\text{Work done per unit mass} = V_w u \pm V_{w1} u_1 \quad \dots[22.34(a)]$$

It will give work done in N-m/kg

Sometimes, the work done is expressed as

$$\text{Work done per unit weight} = \frac{1}{g} (V_w u \pm V_{w1} u_1) \quad [22.34(b)]$$

If the unit weight is newton, the work done per unit newton is in N-m/N.

If the unit weight is kilonewton, the work done per unit kilonewton is in kN-m/kN.

Work done by a weight  $W$  of fluid per second

$$= \frac{W}{g} (V_w u \pm V_{w1} u_1) \quad \dots[22.34(c)]$$

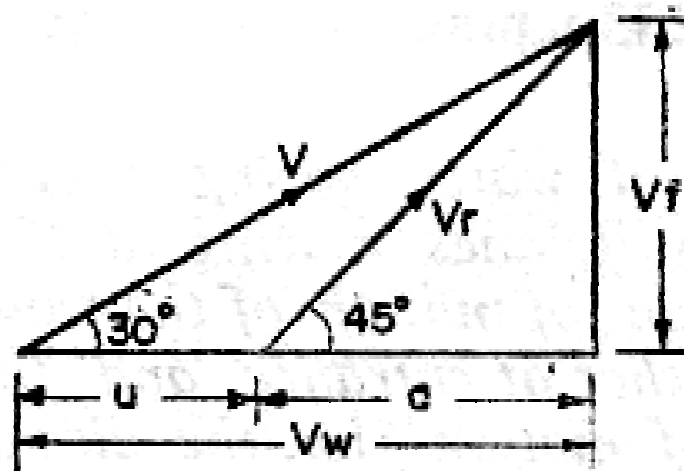
If  $W$  is in newton/sec, the workdone is in N-m/sec (Watts).  
However, if  $W$  is in kilonewton/sec, the workdone is in kN-m/sec (kilowatts).

Of course,  $W = \gamma Q = \rho g Q$

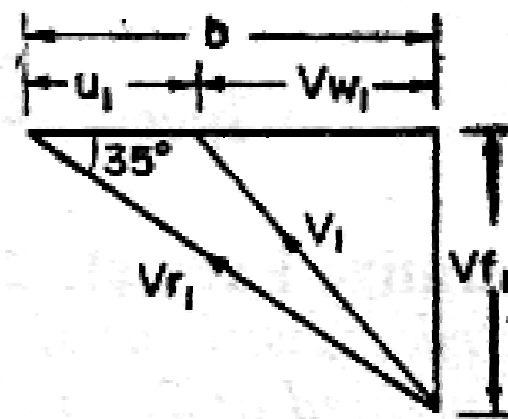
where  $Q$  is the discharge and  $\rho = 1000 \text{ kg/m}^3$  and  $\gamma = 9810 \text{ N/m}^3$  ( $9.81 \text{ kN/m}^3$ ) for water.

### **Problem:**

A wheel having radial blades has the inner and outer radii of 30 cm and 60 cm respectively, The jet enters the blades at the outer tip with a velocity of 40 m/sec at an angle of  $30^\circ$  to the tangent and leaves the blades with a velocity of flow of 8 m/sec. If the angles of the blades at entrance and exit are respectively  $45^\circ$  and  $35^\circ$ , **find the work done per kg of water entering the wheel, the speed of the wheel and its efficiency.**



INLET VELOCITY TRIANGLE



OUTLET VELOCITY TRIANGLE

From the inlet velocity triangle,

$$V_f = V \sin 30^\circ = 40 \times 0.50 = 20 \text{ m/sec}$$

$$V_w = V \cos 30^\circ = 40 \times \frac{\sqrt{3}}{2} = 34.64 \text{ m/sec}$$

$$\text{Velocity 'a'} = \frac{V_f}{\tan 45^\circ} = V_f = 20 \text{ m/sec}$$

Thus

$$\begin{aligned} u &= V_w - a \\ &= 34.64 - 20 = 14.64 \text{ m/sec} \end{aligned}$$

Now

$$\frac{u}{u_1} = \frac{R}{R_1} = \frac{60}{30} = 2$$

Therefore,

$$u_1 = \frac{1}{2}u = 7.32 \text{ m/sec}$$

### From the outlet velocity triangle,

$$\text{Velocity 'b'} = \frac{V_{f1}}{\tan 35^\circ} = \frac{8}{0.7} = 11.43 \text{ m/sec}$$

$$\begin{aligned} V_{w1} &= b - u_1 \\ &= 11.43 - 7.32 = 4.11 \text{ m/sec.} \end{aligned}$$

As the component  $V_{w1}$  is in the negative direction, plus sign will be taken in Eq. (22.34). Thus

$$\begin{aligned} \text{Work done} &= M(V_w u + V_{w1} u_1) \\ &= 1(34.64 \times 14.64 + 4.11 \times 7.32) \text{ per unit mass} \\ &= 537.21 \text{ N-m/kg} \end{aligned}$$

Efficiency of the wheel

$$\begin{aligned} &= \frac{\text{Work done}}{\text{Input}} = \frac{537.21}{\frac{1}{2} \times 1 \times 40 \times 40} = 0.6715 \\ &= 67.15\%. \end{aligned}$$

Angular speed of the wheel

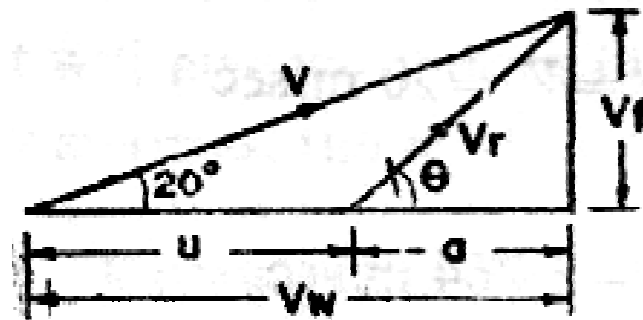
$$\omega = \frac{u}{R} = \frac{14.64}{0.60} = 24.4 \text{ radians/sec}$$

$$\text{Speed} = \frac{24.4 \times 60}{2\pi} = 233 \text{ r.p.m.}$$

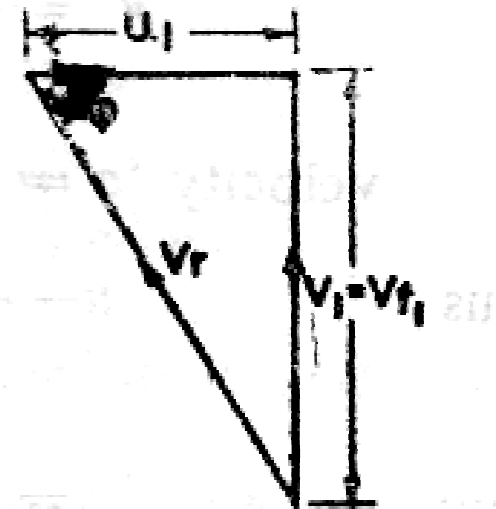
## Problem

A jet of water having a velocity of 50 m/sec strikes, without shock, a series of vanes moving at 15 m/sec. The jet is inclined at  $20^\circ$  to the direction of motion of the vanes. If the relative velocity at exit is 0.9 times that at entrance and the velocity at exit is normal to the direction of motion (i.e. the discharge is radial), find the vane angles at entrance and exit and the efficiency of the wheel.

## Solution:



INLET TRIANGLE



OUTLET TRIANGLE

## From the inlet triangle:

$$V_w = V \cos 20^\circ = 50 \times 0.94 = 46.98 \text{ m/sec}$$

$$V_f = V \sin 20^\circ = 50 \times 0.342 = 17.1 \text{ m/sec}$$

$$\text{Velocity 'a'} = V_w - u = 46.98 - 15.0 = 31.98$$

$$\tan \theta = \frac{V_f}{a} = \frac{17.1}{31.98} = 0.534 \text{ or } \theta = 28.13^\circ$$

$$\text{Also } V_r = \frac{V_f}{\sin 28.13^\circ} = \frac{17.10}{0.471} = 36.3 \text{ m/sec}$$

$$\text{Now } V_{r1} = 0.9 V_r = 0.9 \times 36.3 = 32.67 \text{ m/sec.}$$

Taking  $u = u_1$  from the outlet triangle,

$$\cos \phi = \frac{u_1}{V_{r1}} = \frac{15}{32.67} = 0.459$$

$$\phi = 62.67^\circ.$$

## Efficiency of the wheel,

$$\eta = \frac{M(V_w u \pm V_{w1} u_1)}{MV^2/2}$$

$$= 2 (V_w u \pm V_{w1} u_1) / V^2$$

$$= 2 [(46.98 \times 15) - (0 \times 15)] / 50^2$$

$$= 0.56 \approx 56\%$$

$$u = 15 \text{ m/sec} = u_1$$

$$V_w = 46.98 \text{ m/sec}$$

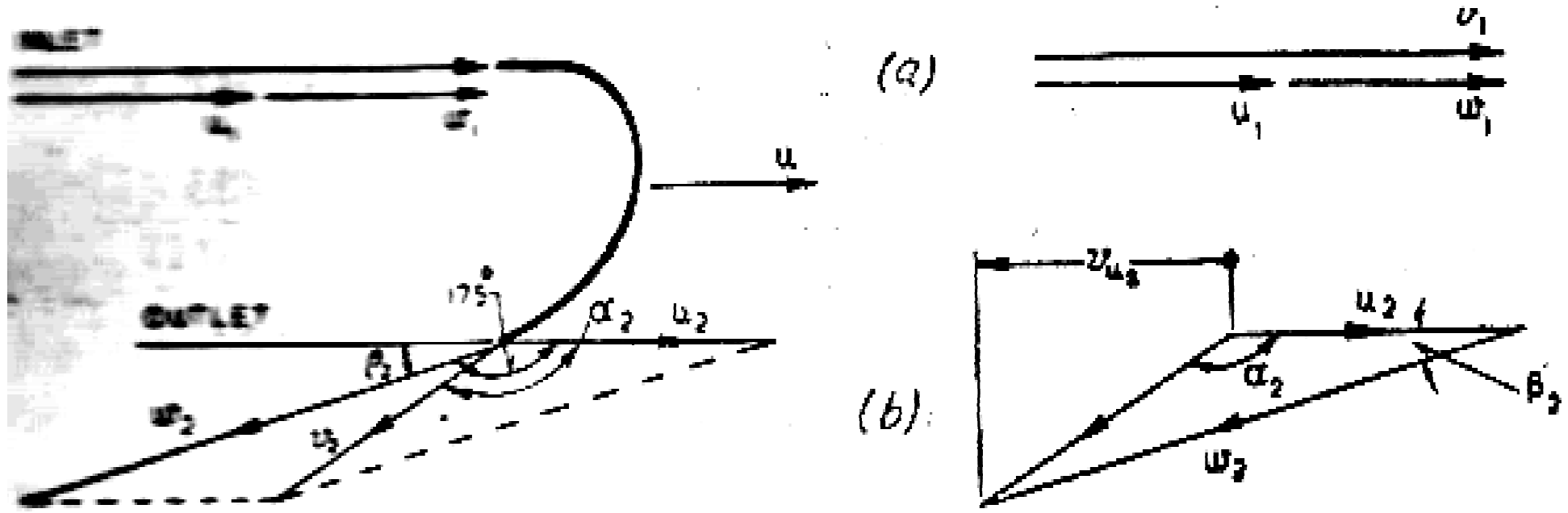
$$V_{w1} = 0 \text{ m/sec and}$$

$$V = 50 \text{ m/sec}$$

### Problem:

A jet of water 5 cm in diameter impinges on a curved vane and is deflected, through an angle of  $175^\circ$ . The vane moves in the same direction as that of the jet with a velocity of 35 m/sec. The rate of flow is 170 lit/sec.

Determine the component of force on the vane in the direction of motion. How much would be the Horse Power developed by the vane and, what would be the water efficiency? Neglect friction.



$$d = 5 \text{ cm} \quad u = 35 \text{ m/sec} \quad Q = 170 \text{ lit/sec} = 0.17 \text{ m}^3/\text{sec}$$

$$v_1 = \frac{Q}{a} = \frac{0.17}{\frac{\pi}{4} \times \left(\frac{5}{100}\right)^2} = 86.8 \text{ m/sec}$$

Assume  $\alpha_1 = 0$  and  $\beta_1 = 0$ ;  $\beta_2 = 180^\circ - 175^\circ$  (refer Fig 1.15b)

Velocity of jet in direction of motion at inlet,  $v_1 \cos \alpha_1 = v_{u_1} = v_1$   
and velocity of jet in direction of motion at outlet,  $v_2 \cos \alpha_2 = v_{u_2}$

Considering the inlet velocity triangle first—

$$v_1 = u_1 + w_1$$

$$\therefore w_1 = v_1 - u_1 = 86.8 - 35 = 51.8 \text{ m/sec.}$$

Now  $w_1 = w_2$ , assuming that no loss occurs along the vane from inlet to outlet. Further the vane is moving with a velocity of 35 m/sec. The radius of arc of a circle drawn from the centre of the wheel to the point where the jet impinges and to the point where the jet leaves the vane, is the same, therefore  $u = \omega r = u_1 = u_2$ .

From outlet velocity triangle (refer Fig 1.15b)

$$\begin{aligned} v_2 \cos \alpha_2 = v_{u_2} &= u_2 - w_2 \cos \beta_2 \\ &= 35 - 51.8 \cos 5^\circ \\ &= 35 - 51.8 \times 0.9962 = 35 - 51.6 \\ &= -16.6 \text{ m/sec} \end{aligned}$$

## Force exerted by the jet on the vane in the direction of motion

$$\begin{aligned}F_u &= \rho \cdot a \cdot (v_1 - u_1)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \\&= \frac{1,000}{9.81} \times \frac{\pi}{4} \times \left(\frac{5}{100}\right)^2 \times 51.8 \times (86.8 + 16.6) \\&= \mathbf{1,075 \text{ kg}} \text{ Answer}\end{aligned}$$

## HP developed by jet

$$= \frac{F_u \cdot u}{75} = \frac{1,075 \times 35}{75} = \mathbf{502 \text{ HP}} \text{ Answer}$$

## Water efficiency

$$\begin{aligned}&= \frac{\text{HP developed} \times 75}{\text{K.E. of jet } \left(\frac{1}{2} m \cdot v_1^2\right)} \\&= \frac{502 \times 75}{\frac{1}{2} \times \frac{1,000 \times 0.17}{9.81} \times 86.8^2} \\&= \mathbf{0.575 \text{ or } 57.5\%} \text{ Answer}\end{aligned}$$

(b) A series of vanes fixed to a wheel, instead of one vane only

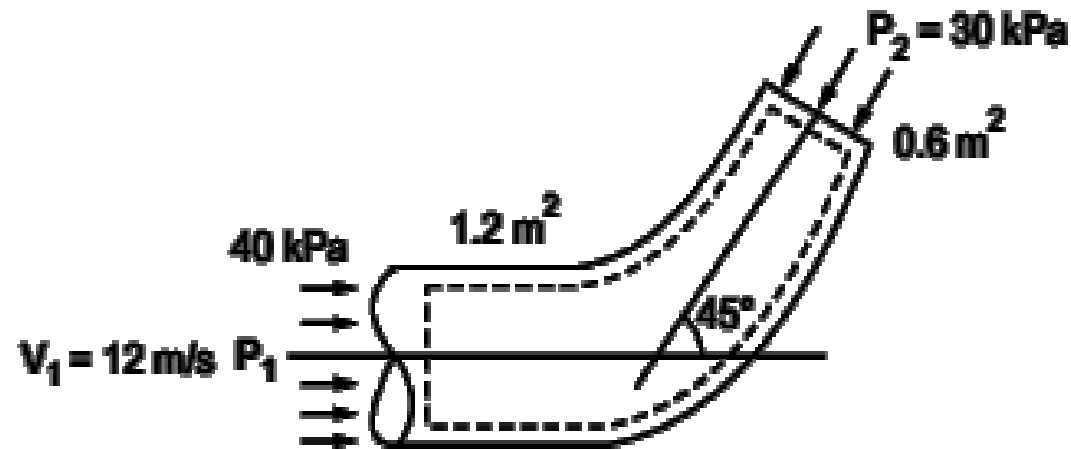
$$\begin{aligned}F_u &= \rho (a \cdot v_1)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \\&= \frac{1,000}{981} \times 0.17 \times (86.8 + 16.6) \\&= 1,790 \text{ kg} \quad \text{Answer}\end{aligned}$$

$$\text{HP developed by jet} = \frac{F_u \cdot u}{75} = \frac{1,790 \times 35}{75} = 835 \text{ HP} \quad \text{Answer}$$

$$\begin{aligned}\text{Water efficiency} &= \frac{\text{HP developed}}{\text{K.E. of jet}} \times 75 = \frac{835 \times 75}{\frac{1}{2} \times \frac{1,000 \times 0.17}{9.81} \times 86.8^2} \\&= 0.959 \text{ or } 95.9\% \quad \text{Answer}\end{aligned}$$

**Problem:**

A  $45^\circ$  bend in the horizontal plane is shown in figure. The inlet area is  $1.2 \text{ m}^2$  and the outlet area is  $0.6 \text{ m}^2$ . The velocity of water at inlet is  $12 \text{ m/s}$ . The pressures at inlet and outlet are  $40$  and  $30 \text{ kPa}$  respectively. **Calculate the magnitude and direction of the resultant force on the bend.**



**Note:** For convenience the control volume should be chosen such that the inlet and outlet areas are normal to the velocities at these sections. In this case the force on the bend is required. It is convenient to calculate the forces in the  $x$  and  $y$  directions separately.

**Solution:**

*Given*  $u_1 = 12 \text{ m/s}$       since  $Q = A_1 u_1 = A_2 u_2 = 1.2 \times 12 \text{ m}^3/\text{s} = 0.6 \times u_2$   
 $\therefore u_2 = 24 \text{ m/s}$

Mass flow =  $12 \times 1.2 \times 1000 = 14.4 \times 10^3 \text{ kg/s}$

Using equations (13.1.7) and (13.1.8)