ME265: Thermal Engineering & Heat Transfer

Chapters
1. Energy Scenario
2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

4.1 Introduction	
4.2 Conduction	
4.3 **Convection**	**4.3.1 Convection Fundamentals** **4.3.2 External Forced Convection** **4.3.3 Internal Forced Convection** **4.3.4 Natural Convection**
4.4 Radiation	
4.5 Heat Exchanger	

4.3.3 Internal Forced Convection

❑ Objectives:

(i) Review of Internal flows:

- **Average velocity** and **average temperature** from the knowledge of velocity and temperature profiles in internal flows
- Laminar and turbulent flows in tubes
- **Hydrodynamic and thermal boundary layers**
- Visual understanding of different **flow regions in internal flow**, such as the entry and fully developed flow regions

(ii) General Thermal Analysis:

- Analyze heating/cooling of a fluid flowing in a tube under **constant surface temperature** and **constant surface heat flux** conditions
- Work with the logarithmic mean temperature difference **(LMTD)**

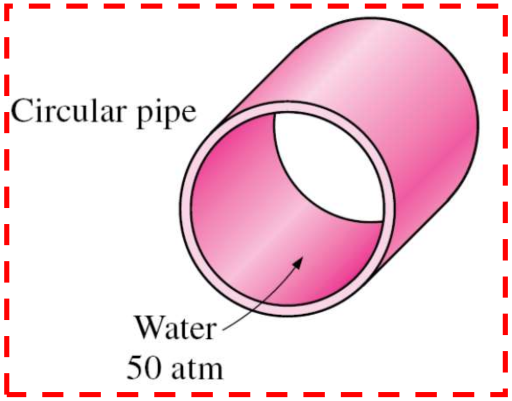
(iii) Forced Convection in Tubes:

- Obtain analytic relations for **Nusselt number/heat transfer rate** in **fully developed laminar flow.**
- Use **empirical relations** to determine **Nusselt number/heat transfer rate** in the fully developed turbulent flow

4.3.3 Internal Forced Convection

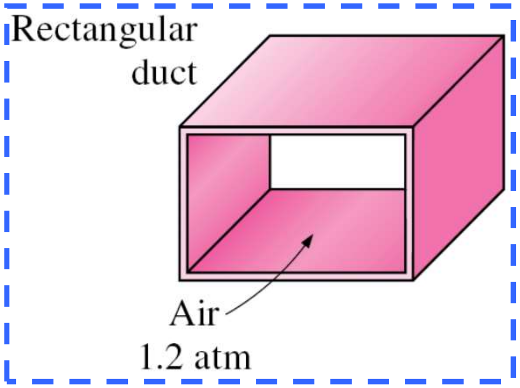
Internal flow fundamentals

❑ Classification of flow channels:



- **Pipe** — circular cross section.
- **Tubes** — small-diameter pipes.

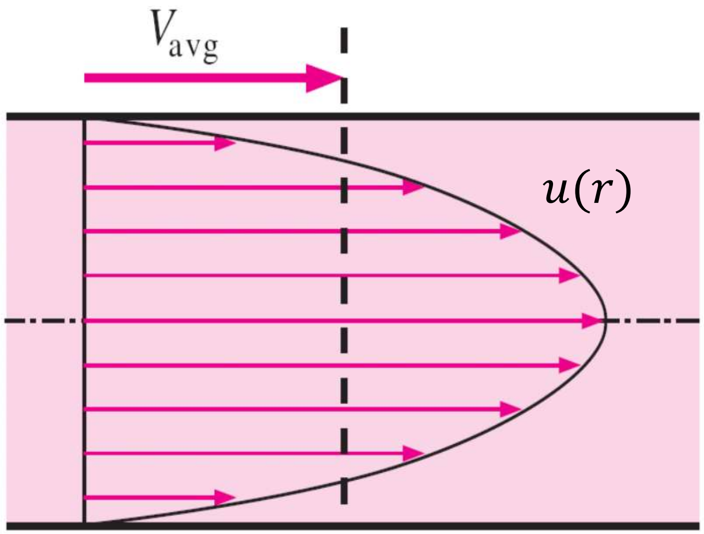
- **Duct** — noncircular cross section.
- **Conduit**—circular / noncircular cross-section to convey hot gases



4.3.3 Internal Forced Convection

❑ Average velocity, V_{avg}

- The fluid velocity changes from zero at the surface (no-slip) to a maximum at the pipe center.
- It is convenient to work with an average velocity, V_{avg}, which remains constant in incompressible flow in a constant cross-sectional pipe.



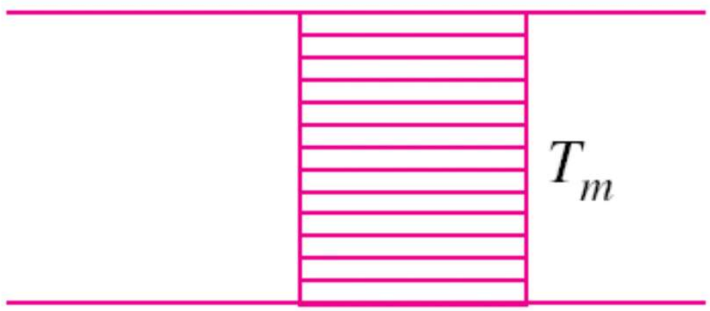
$$\dot{m} = \rho V_{avg} A_C = \int_{A_c} \rho u(r)\, dA_C$$

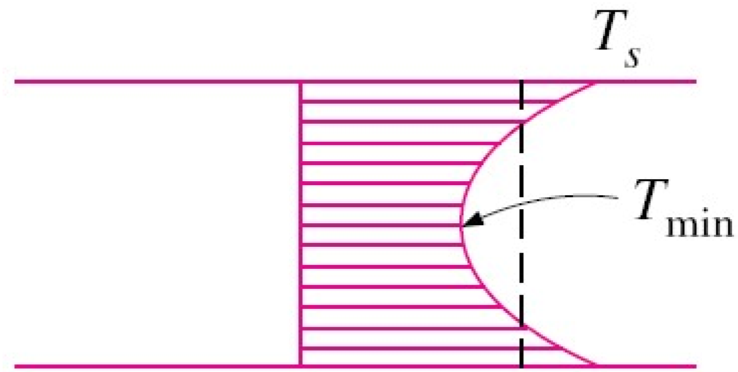
For incompressible flow in a circular pipe of radius *R*

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_C}{\rho A_C} = \frac{\int_0^R \rho u(r) 2\pi r dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r dr \quad \text{...... (3.3.1)}$$

4.3.3 Internal Forced Convection

❑ Average / mean Temperature, T_m





$$\dot{E}_{fluid} = \dot{m} c_p T_m = \int_{\dot{m}} c_p T(r) \delta \dot{m} = \int_{A_c} \rho c_p T(r) u(r) V dA_c$$

For incompressible flow in a circular pipe of radius *R*

$$T_m = \frac{\int_{\dot{m}} c_p T(r) \delta \dot{m}}{\dot{m} c_p} = \frac{\int_{A_c} c_p T(r) \rho u(r) 2\pi r dr}{\rho V_{avg} \left(\pi R^2\right) c_p}$$

$$= \frac{2}{V_{avg} R^2} \int_0^R T(r) u(r) r dr \qquad \text{…… (3.3.2)}$$

EP# 3.7

The velocity and temperature profile for a fluid flowing in a circular tube of radius, R= 4 cm are given by: $u(r) = 0.2[1 - (r/R)^2]$ in m/s and $\mathrm{T}(r) = 250 + 200(r/R)^3$ in K
Determine the mean fluid temperature.

4.3.3 Internal Flow Convection: Fundamentals

❑ Laminar and Turbulent Flow in Tubes

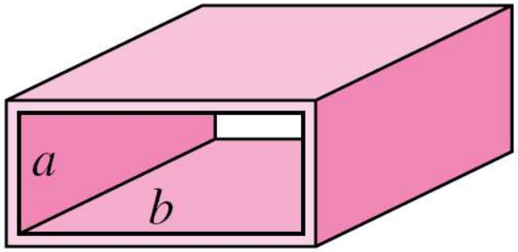
- For flow in a circular tube, the Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{V_{avg} D}{\nu}$$

- For flow through noncircular tubes D is replaced by the hydraulic diameter D_h.

$$D_h = \frac{4 A_c}{P}$$

Rectangular duct:

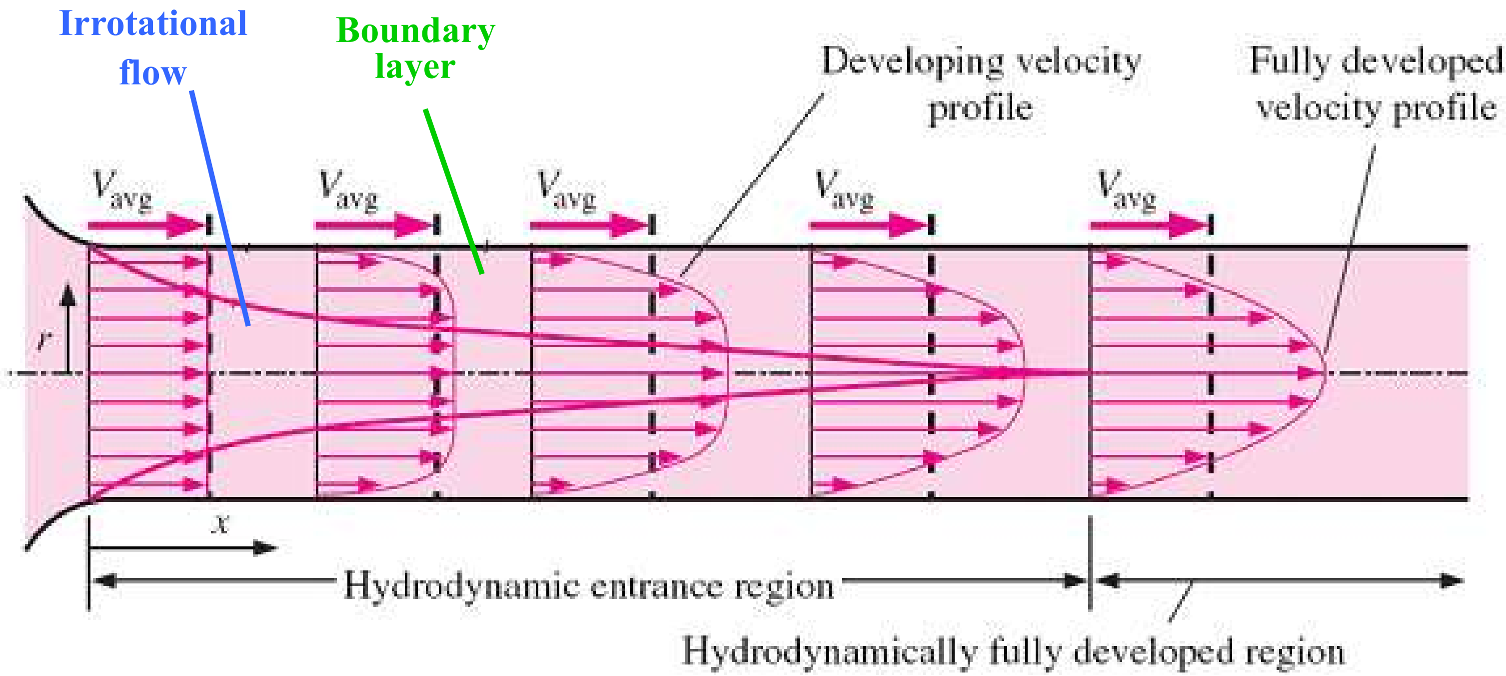


$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

- Laminar flow : Re < 2300
- Transitional flow : 2300 < Re < 10,000
- Fully turbulent flow : Re > 10,000

4.3.3 Internal Flow Convection: Fundamentals

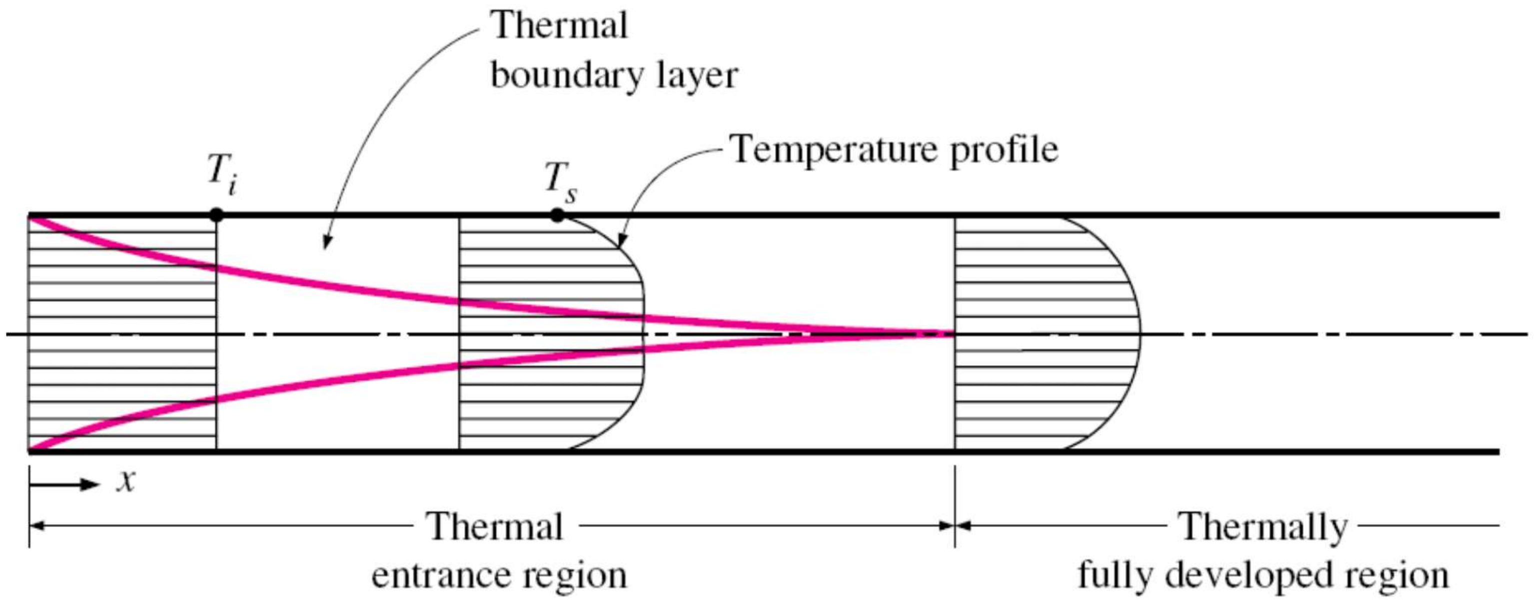
❑ Hydrodynamic Entrance Region



- The thickness of this boundary layer increases in the flow direction until it reaches the pipe center.
- **Hydrodynamic entrance region —** the region from the pipe inlet to the point at which the boundary layer merges at the centerline.
- **Hydrodynamically fully developed region —** the region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

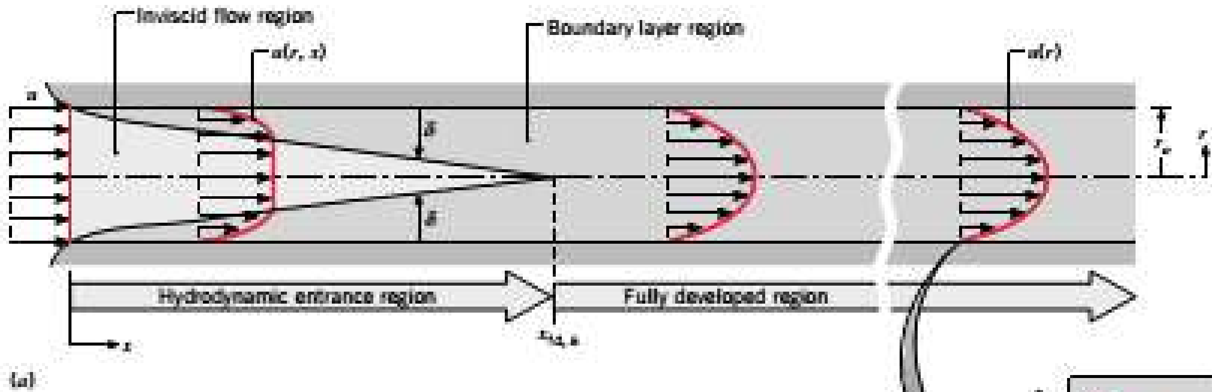
4.3.3 Internal Flow Convection: Fundamentals

❑ Thermal Entrance Region



- The thickness of this boundary layer increases in the flow direction until it reaches the pipe center.
- **Thermal entrance region —** the region from the pipe inlet to the point at which the thermal boundary layer merges at the centerline.
- **Thermally fully developed region —** the region beyond the entrance region in which the temperature profile is fully developed and remains unchanged.

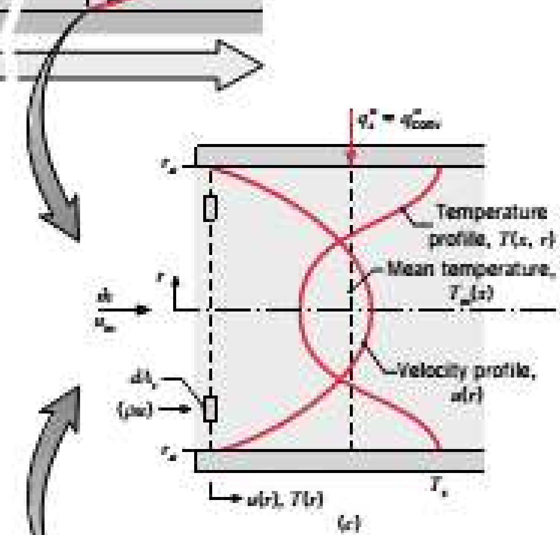
❑ Summary of Convection flow regions

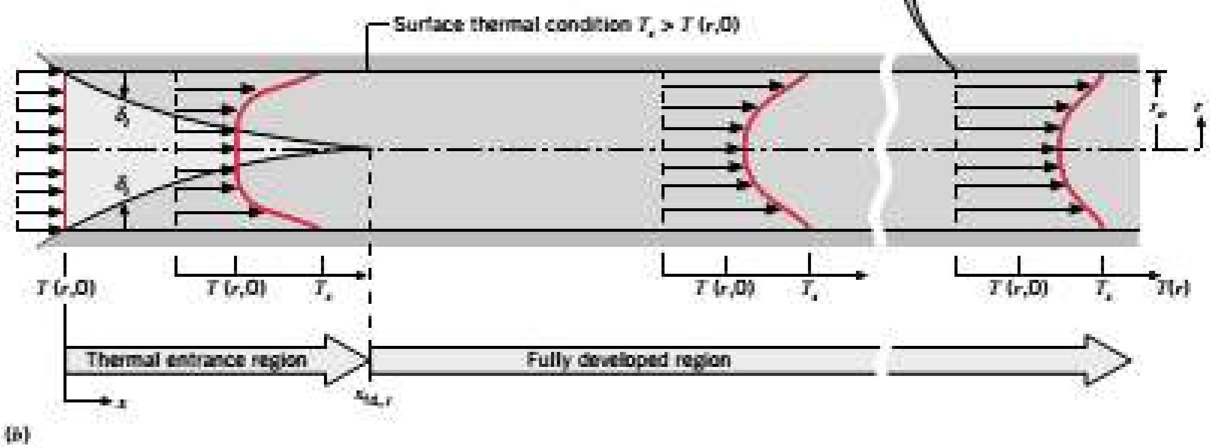


$$L_{h,\text{laminar}} \approx 0.05\,\text{Re}\cdot D$$

$$L_{t,\text{laminar}} \approx 0.05\,\text{Re}\cdot\text{Pr}\cdot D = \text{Pr}\cdot L_{h,\text{laminar}}$$

$$L_{h,turbulent} \approx L_{t,turbulent} \approx 10D$$





4.3.3 Internal Flow Convection: Fundamentals

❑ Characteristics of fully developed flow

- Hydrodynamically fully developed:

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

- Thermally fully developed:

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=R} = \frac{-(\partial T/\partial r)\big|_{r=R}}{T_s - T_m} \neq f(x)$$

- Surface heat flux can be expressed as

$$\dot{q}_s = h_x (T_s - T_m) = k\frac{\partial T}{\partial r}\bigg|_{\mathrm{r=R}} \rightarrow h_x = \frac{k(\partial T/\partial r)\big|_{r=R}}{T_s - T_m}$$

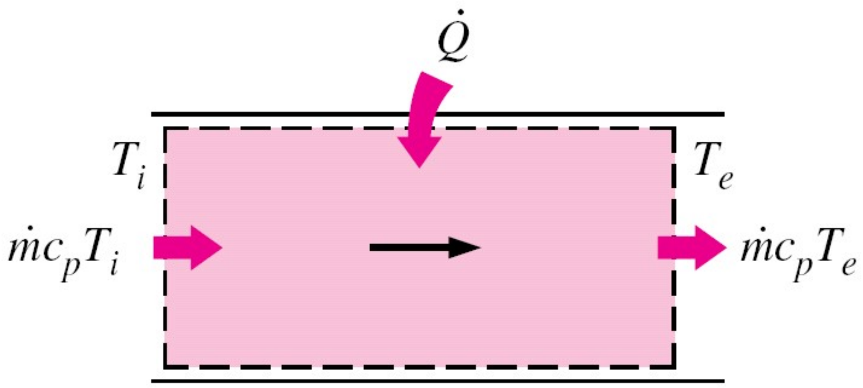
$$\Rightarrow \quad h_x \neq f(x)$$

$$h_x = \text{constant}$$

4.3.3 Internal Forced Convection

❑ General Thermal Analysis (GTA)

In the absence of any work interactions, the conservation of energy equation for the steady flow in a tube is given by:



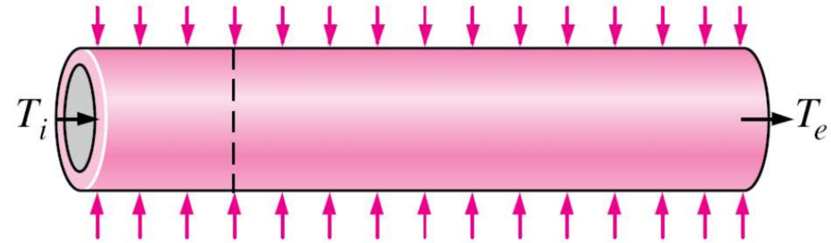
$$\dot{Q} = \dot{m}c_p\left(T_e - T_i\right) \qquad \text{… … (3.3.3)}$$

Where, T_i and T_e are the mean fluid temperatures at inlet and exit

- The thermal conditions at the surface can usually be approximated as:
  - **constant surface temperature**
  - **constant surface heat flux**.

❑ GTA for Constant surface heat flux

- The rate of heat transfer can be expressed as



$$\dot{Q} = \dot{q}_s A_s$$

$$\dot{Q} = \dot{m} c_p \left(T_e - T_i \right) \qquad \text{… … (3.3.3)}$$

- Then the mean fluid temperature at the tube exit becomes

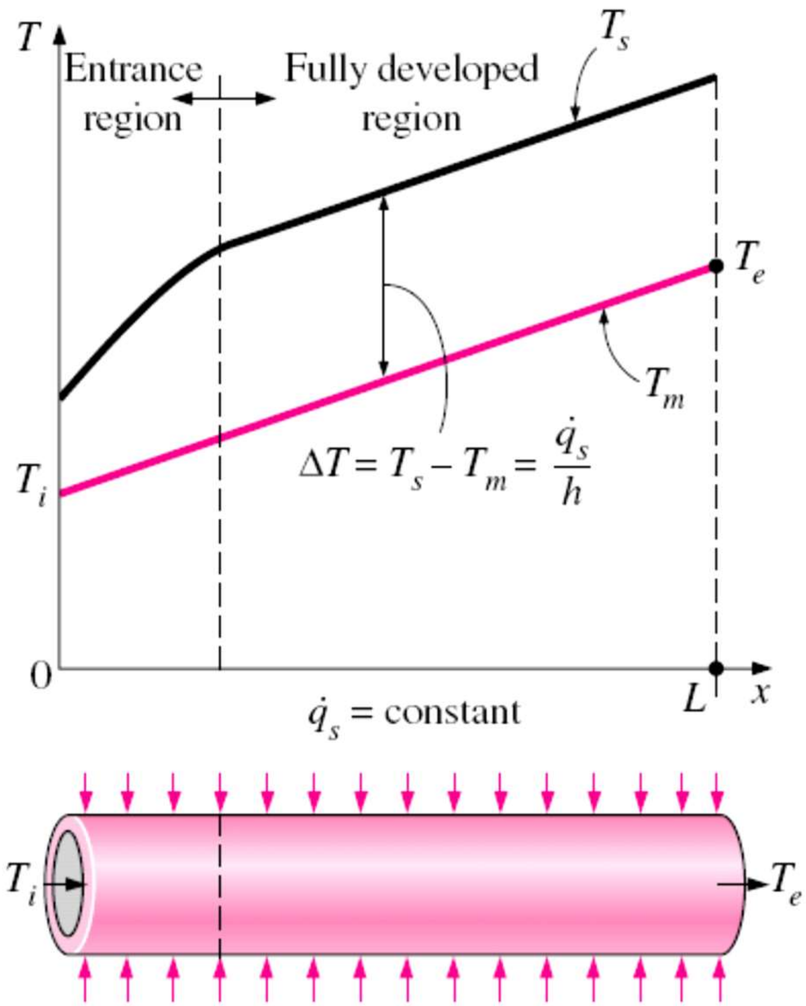
$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} c_p} \qquad \text{… … (3.3.4)}$$

- Again, the surface temperature can be determined from

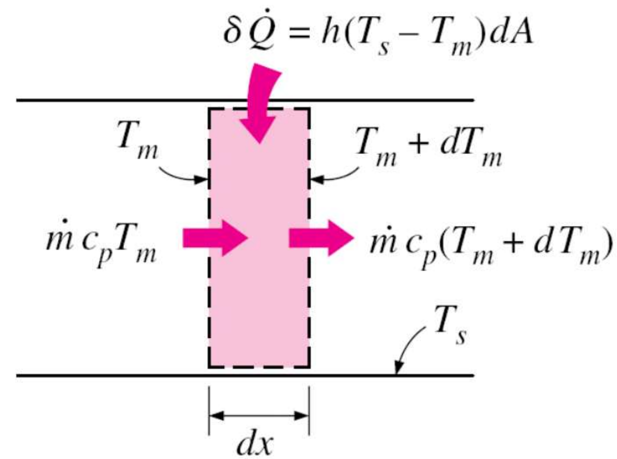
$$\dot{q}_s = h\left(T_s - T_m \right) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h} \qquad \text{… … (3.3.5)}$$

$$\Rightarrow \quad \frac{dT_s}{dx} = \frac{dT_m}{dx}$$

❑ GTA for Constant surface heat flux



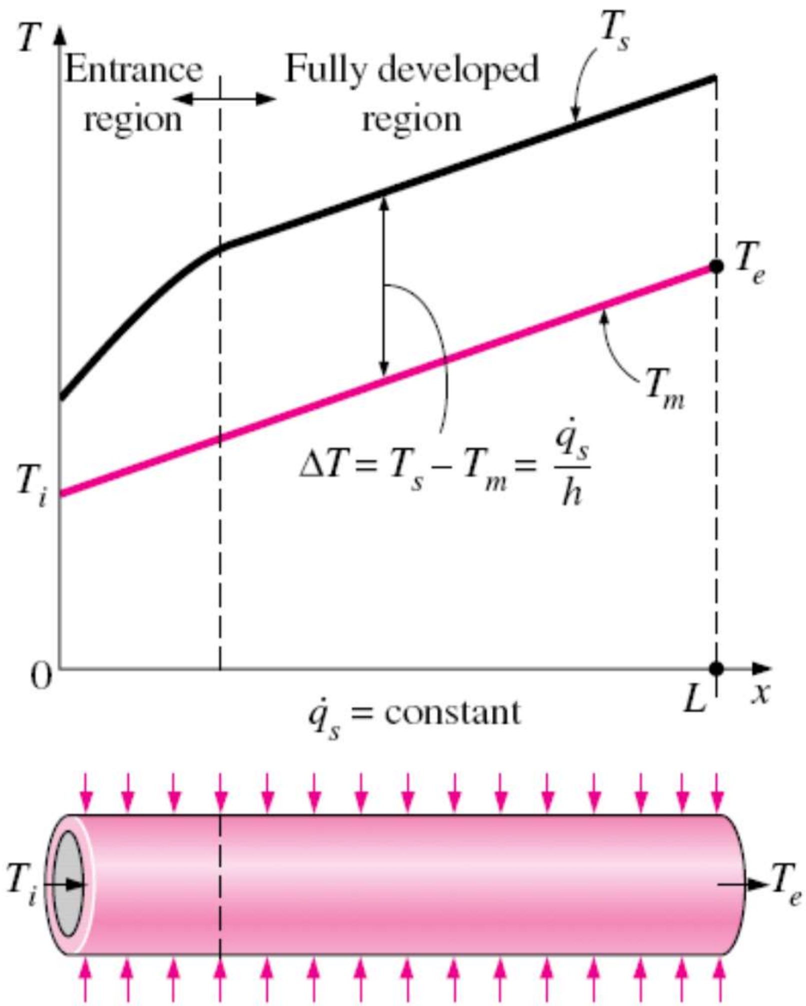
$$\frac{dT_s}{dx} = \frac{dT_m}{dx}$$



$$\dot{m}c_p dT_m = \dot{q}_s\left(p dx\right)$$

$$\rightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant}$$

❑ GTA for Constant surface heat flux



$$\frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = constant$$

In the fully developed region:

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$

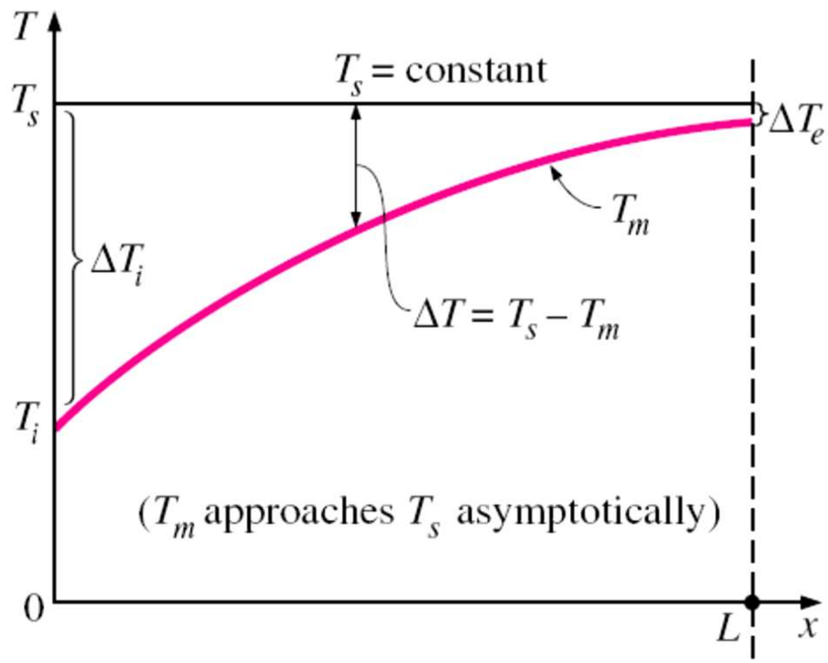
$$\rightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0$$

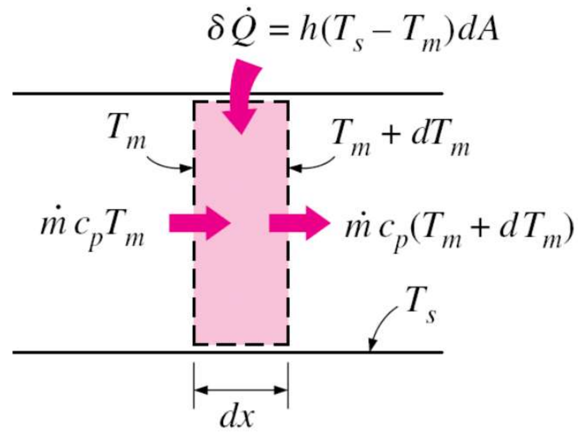
$$\rightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

Therefore,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = constant$$

❑ GTA for Constant surface temperature





Energy Balance:

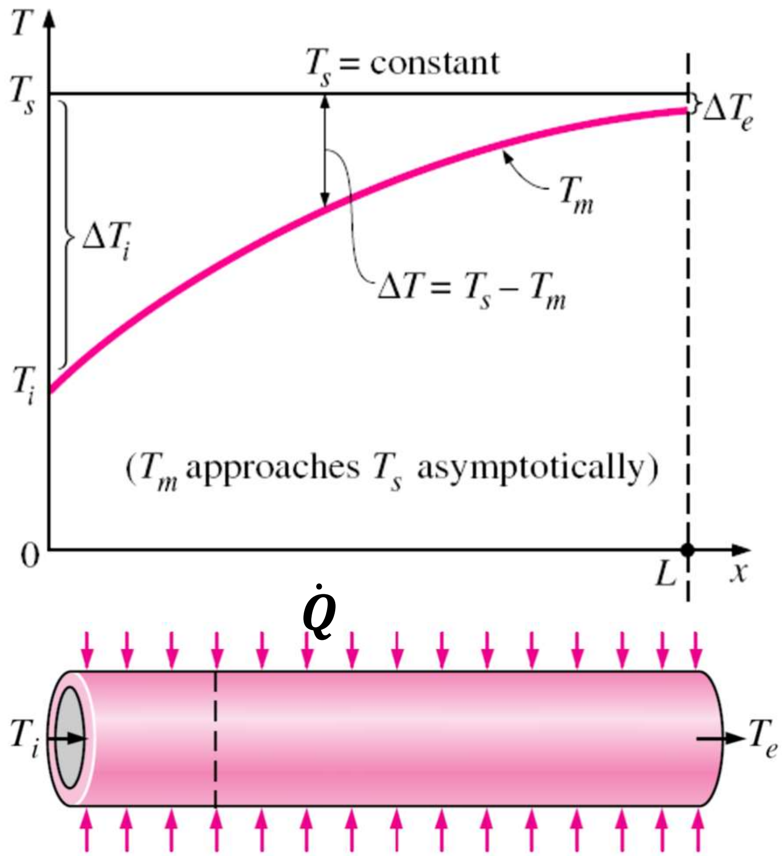
$$\delta \dot{Q} = \dot{m} c_p dT_m = h\left(T_s - T_m\right) dA_s$$

$$\Rightarrow \frac{d(T_s - T_m)}{(T_s - T_m)} = \frac{h\, dA_s}{\dot{m}\, c_p}$$

- Integrating from x=0 (at inlet where T_m=T_i) to x=L (at exit where T_m=T_e) gives

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} c_p} \qquad \text{…… (3.3.6)}$$

❑ GTA for Constant surface temperature



$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p} \qquad \text{… … (3.3.6)}$$

$$\Rightarrow \dot{m}c_P = -\frac{hA_s}{ln[(T_s - T_e)/(T_s - T_i)]}$$

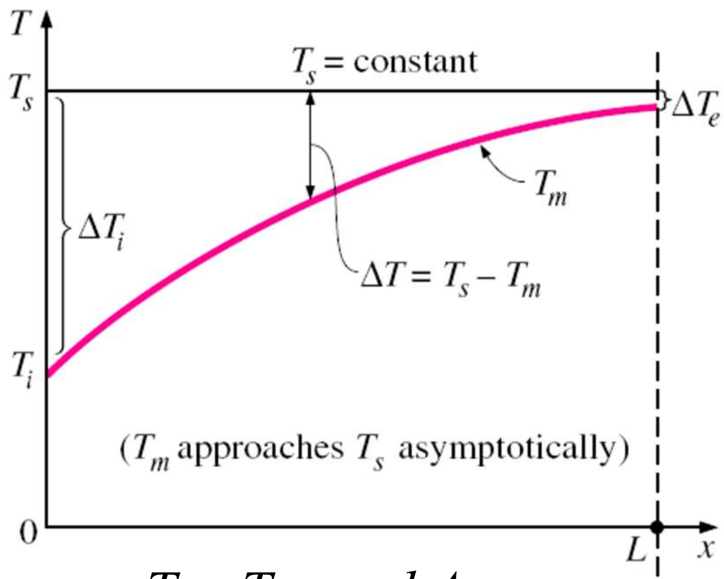
$$\dot{Q} = \dot{m}c_P(T_e - T_i)$$
$$= h\, A_s\, \Delta T_{lm}$$

Here, ΔT_{lm} is the logarithmic mean temperature difference given by:

$$\Delta T_{lm} = \frac{T_i - T_e}{ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{ln[\Delta T_e/\Delta T_i]} \qquad \text{… … (3.3.7)}$$

4.3.3 Internal Forced Convection: GTA

❑ Constant T_s



$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p}$$

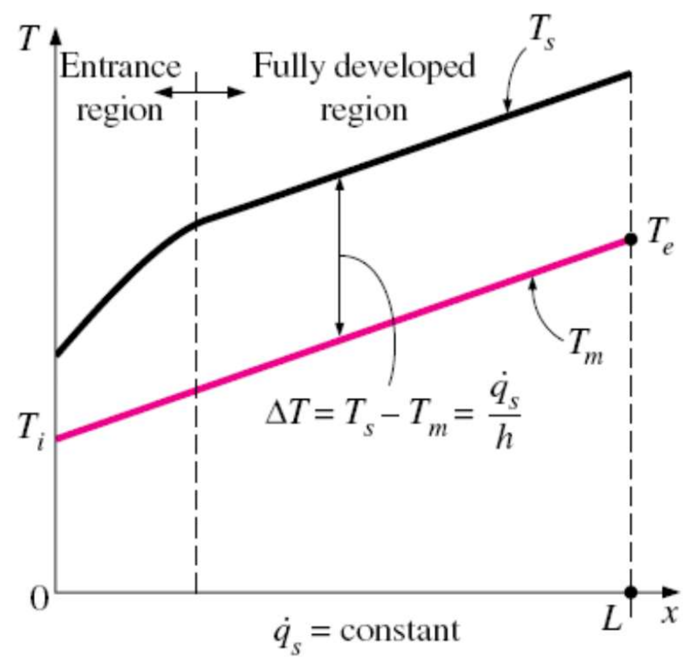
$$\Rightarrow T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right)$$

$$\dot{Q} = h\,A_s\,\Delta T_{lm}$$

$$\Delta T_{lm} = \frac{T_i - T_e}{ln[(T_s - T_e)/(T_s - T_i)]}$$

$$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{ln[\Delta T_e/\Delta T_i]}$$

❑ Constant $\dot{q}_s$



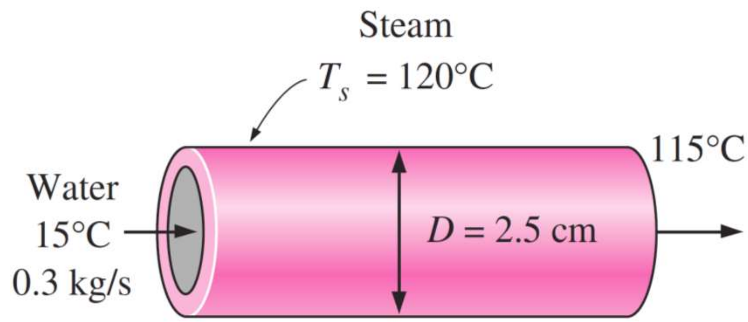
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p}$$

$$\dot{Q} = \dot{m}c_P(T_e - T_i)$$

$$\dot{Q} = h(T_s - T_m)$$

EP# 3.8 Cengel et al. Example: 8-3

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m^{2}C, determine the length of the tube required in order to heat the water to 115°C.



Assumptions:

1. Steady operating conditions exist.
2. Fluid properties are constant.
3. The convection heat transfer coefficient is constant.
4. The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties

c_p=4187 J/kg^{o}C for water at the **bulk mean temperature** of (15 + 115)/2 = 65°C

h_{fg}=2203 kJ/kg for steam at 120°C(Table A-9).