ME265: Thermal Engineering & Heat Transfer

❑ Convection HT coefficient, h:

$$h = \frac{-k\,(\partial T/\partial y)_{y=0}}{(T_s - T_\infty)} \qquad \ldots\ldots (3.1)$$

It is an experimentally determined parameter depending on—

- surface geometry,
- nature of fluid motion
- properties of the fluid: k, ρ, c, μ, &
- bulk fluid velocity: V

❑ For Forced Convection:

$$Nu = f(Re, Pr) \qquad \ldots\ldots (3.2)$$

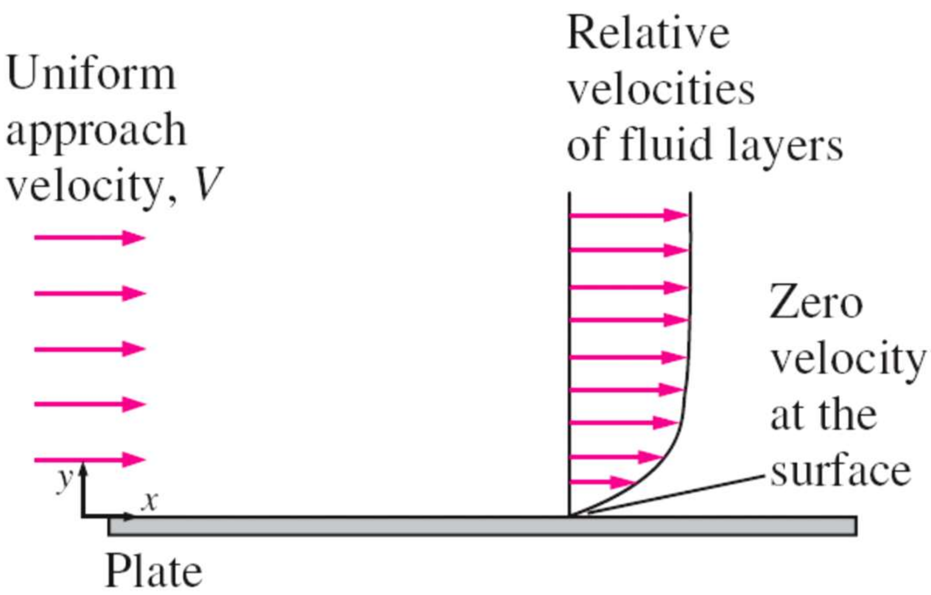
$$Nu = \frac{hL_c}{k} \qquad Re = \frac{\rho\,VL_c}{\mu}$$

$$Pr = \frac{\mu c_p}{k}$$

k is the conductivity of fluid

❑ Characteristics of fluid flows

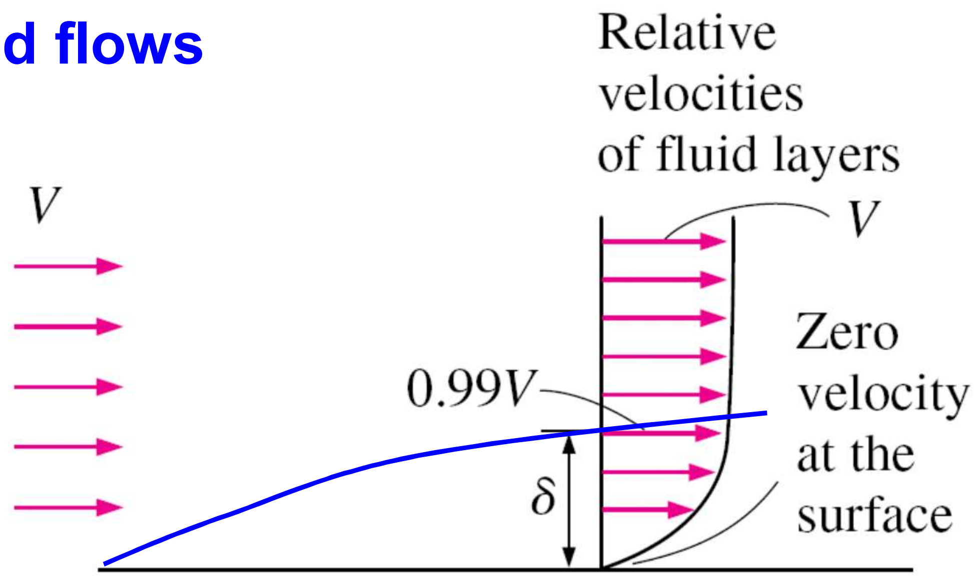
- **Experimental observations**:
  - Fluid in motion comes to a complete stop at the surface
  - It has a zero velocity relative to the surface (**no-slip condition**).



- The motionless layer adjacent to the surface slows down the neighboring fluid layer as a result of friction and causes the development of the velocity profile.

4.3 Convection Heat transfer

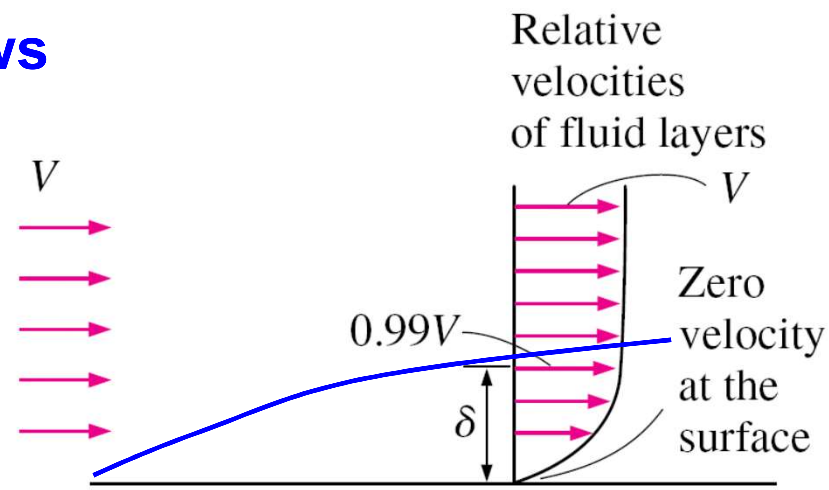
❑ Characteristics of fluid flows



- The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **velocity boundary layer.**
- The fluid velocity, *u,* varies from 0 at *y*=0 to nearly *V* at *y*=δ.
- δ is typically defined as the distance *y* from the surface at which *u*=0.99*V*.
- **This δ is called boundary layer thickness**

4.3 Convection Heat transfer

❑ Characteristics of fluid flows



- The fluid layer in contact with the surface tries to drag the plate exerting a ***friction force*** on it.

- Friction force per unit area is the **shear stress (*τ*).** Experiments indicate that the shear stress for most fluids is proportional to the velocity gradient.
- **Newton's Law of viscosity**: $$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \quad \text{… … (3.7)}$$

- The fluids that follow this linear relationship are called **Newtonian fluids.**

❑ Characteristics of fluid flows

- In many cases the flow velocity profile is unknown and the surface shear stress τ_s can not be obtained.
- A more practical approach in external flow is to relate τ_s to the upstream velocity V as

$$\tau_s = C_f \frac{\rho V^2}{2} \quad \text{…… (3.8)}$$

Where, C_f is the dimensionless **friction coefficient** (most cases it is determined experimentally)

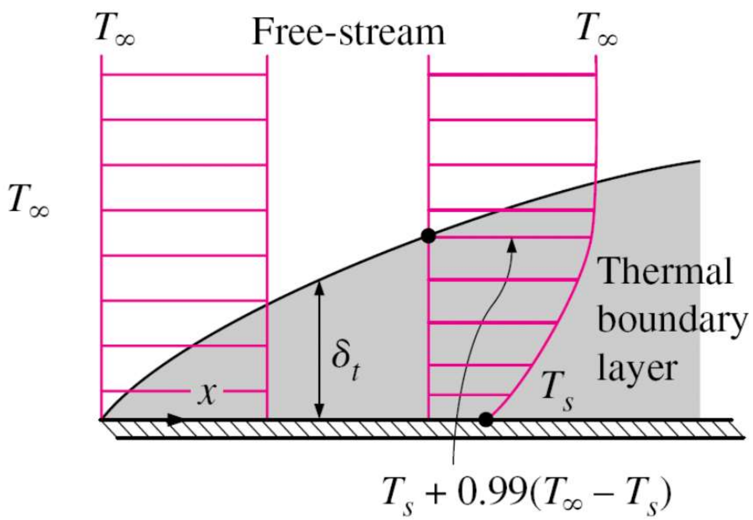
- The friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad \text{…… (3.9)}$$

4.3 Convection Heat transfer

❑ Characteristics of fluid flows

- In flow over a heated (or cooled) surface, both **velocity and thermal boundary layers** develop simultaneously.



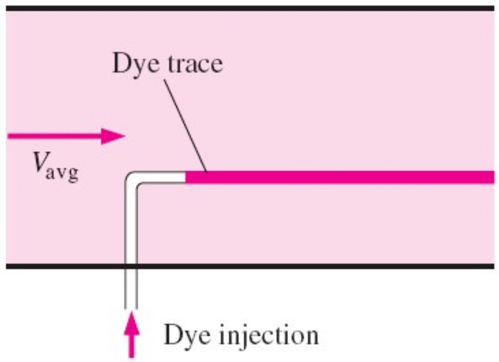
- δ_t is the **thermal boundary layer thickness,** where fluid temperature reaches 99% of the difference between free stream and surface temperatures.
- **Prandtl number, Pr** determines the relative thicknesses of velocity and thermal boundary layers.

➢ Heat diffuses very quickly in liquid metals (Pr<<1) and very slowly in oils (Pr>>1) relative to momentum.

➢ Consequently, the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

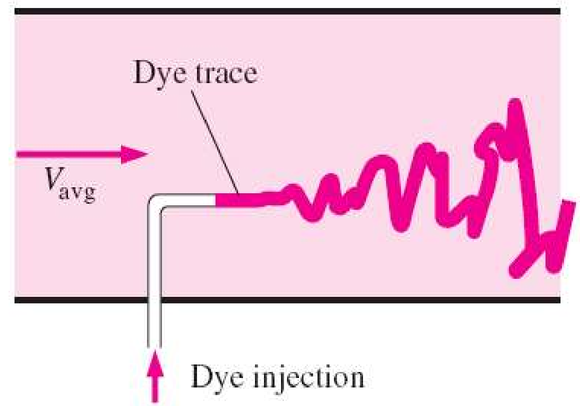
❑ Characteristics of fluid flows

- **Laminar flow and Turbulent flow**



(*a*) Laminar flow

Laminar flow— the flow is characterized by *smooth streamlines* and *highly-ordered motion.*



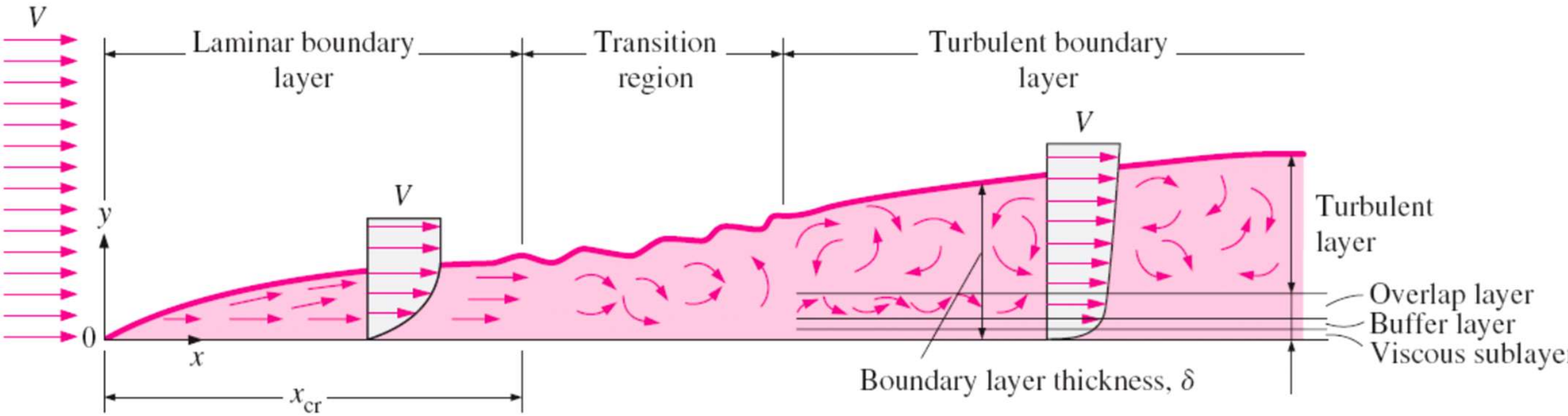
(*b*) Turbulent flow

Turbulent flow — the flow is characterized by *velocity fluctuations* and *highly-disordered motion*

The *intense mixing* in turbulent flow enhances heat and momentum transfer, which increases the friction force on the surface and the convection heat transfer rate.

❑ Characteristics of fluid flows

- **Boundary layer over a flat plate**



The turbulent wall shear stress and turbulent heat transfer

$$\tau_{turb} = -\rho \overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \quad ; \quad \dot{q}_{turb} = \rho c_p \overline{vT} = -k_t \frac{\partial T}{\partial y} \qquad \text{… … (3.10)}$$

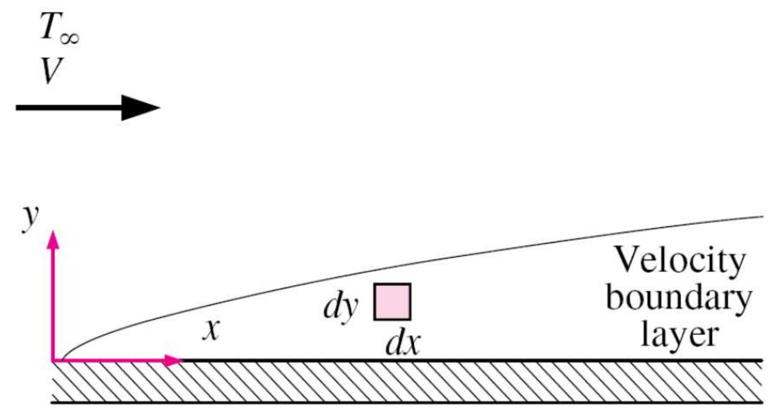
where, μ_t — turbulent (or eddy) viscosity
k_t — turbulent (or eddy) thermal conductivity

4.3.1.5 Governing Equations for forced convection

- ❑ Continuity Equation:
  Conservation of Mass
- ❑ Momentum Equation:
  Conservation of Momentum
- ❑ Energy Equation:
  Conservation of Energy

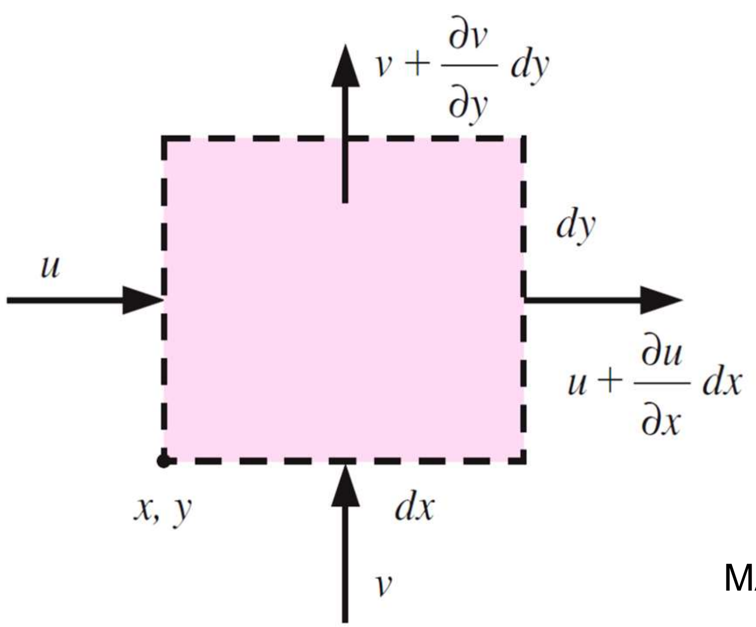
4.3.1.5 Governing Equations for forced convection

❑ Continuity Equation



Assumptions:

- laminar flow over a flat plate
- steady two-dimensional flow
- Newtonian fluid,
- constant properties.

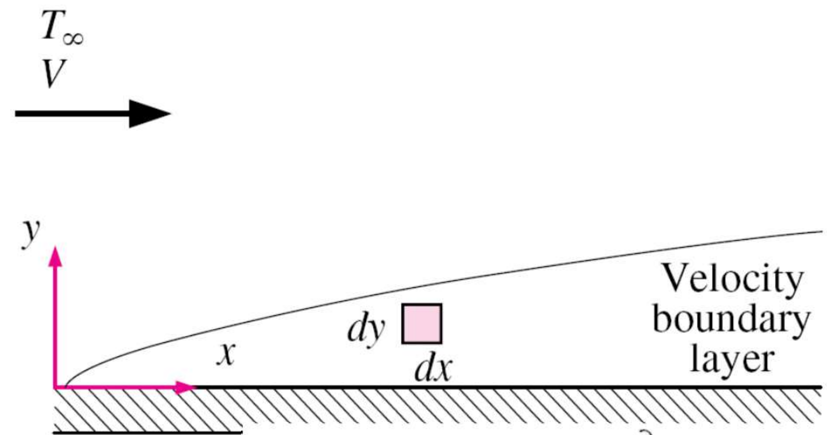


$$\left(\text{Rate of mass flow into the control volume} \right) = \left(\text{Rate of mass flow out of the control volume} \right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{…… (3.11)}$$

4.3.1.5 Governing Equations for forced convection

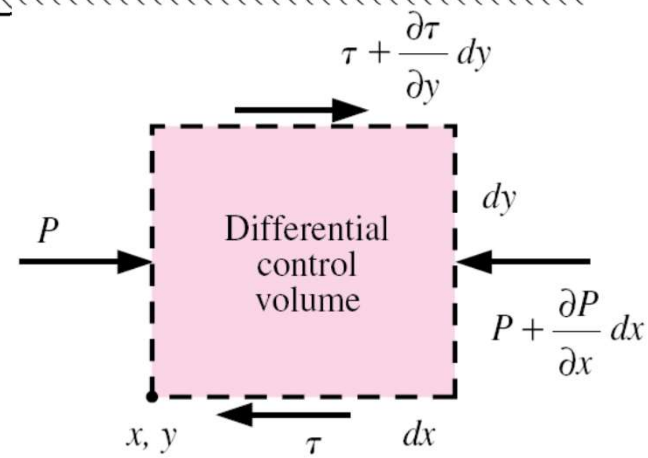
❑ Momentum Equation



Assumptions:

- laminar flow over a flat plate
- steady two-dimensional flow
- Newtonian fluid,
- constant properties.

Newton's 2nd Law:



$$\sum \mathbf{Force}_x = \boxed{\text{Rate of change of momentum in x-direction}}$$

$$\sum \text{Forces}_x = -\frac{\partial P}{\partial x} dx.dy + \frac{\partial \tau}{\partial y} dx.dy$$

$$\Rightarrow \sum \text{Forces}_x = \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right) dx.dy$$

$$\boldsymbol{\tau = \mu \frac{\partial u}{\partial y}}$$

4.3.1.5 Governing Equations for forced convection

Rate of change of momentum in x-direction	=	Mass × Acceleration in x-direction

$$= (\rho\, dx.dy) \times a_x$$

$$= \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) dx.dy$$

Therefore, (3.12) can be simplified to:

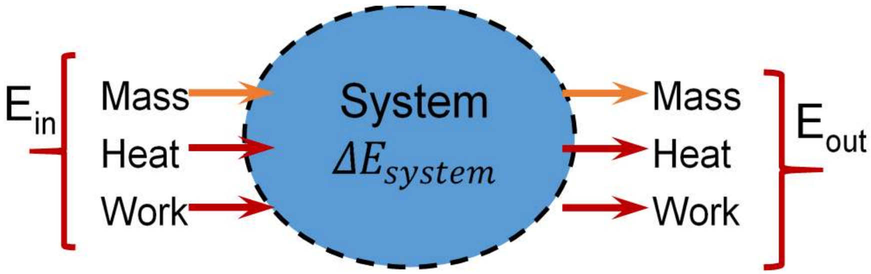
$$\boldsymbol{\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}} \dots\dots (3.12)$$

4.3.1.5 Governing Equations for forced convection

❑ Energy Equation

First Law of Thermodynamics:

$$E_{in} - E_{out} = \Delta E_{system}$$



For steady-state conditions with no energy transfer by work, energy balance is given by:

$$\left(\dot{E}_{out} - \dot{E}_{in}\right)_{Heat} + \left(\dot{E}_{out} - \dot{E}_{in}\right)_{Mass} = 0 \qquad \dots\dots (3.13)$$

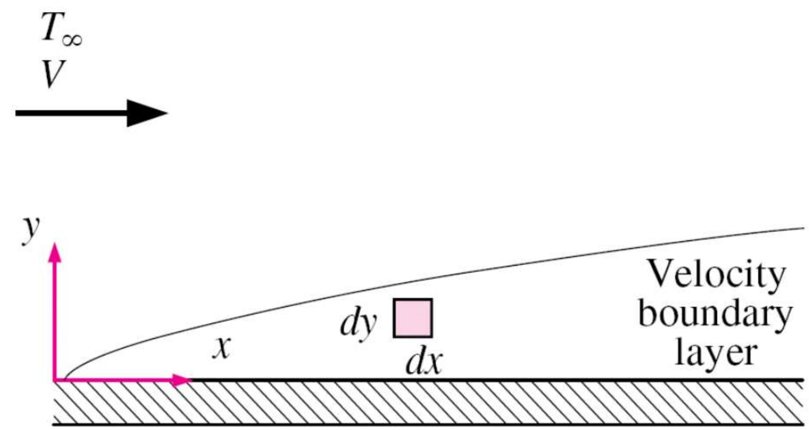
Now, the total rate of Energy of flowing fluid stream is given by:

$$\dot{E} = \dot{m}\,\mathrm{e} = \dot{m}\,(\mathrm{h} + \mathrm{ke} + \mathrm{pe})$$

With negligible kinetic and potential energy, $\dot{E} = \dot{m}\,\mathrm{h} = \dot{m} c_p T$

4.3.1.5 Governing Equations for forced convection

❑ Energy Equation

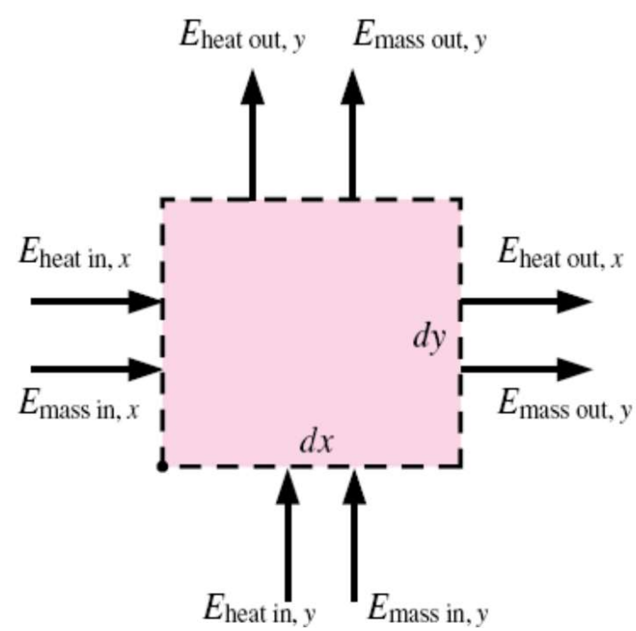


Assumptions

- laminar flow over a flat plate
- steady two-dimensional flow
- Newtonian fluid with constant properties.
- Negligible viscous dissipation

$$\left(\dot{E}_{out}-\dot{E}_{in}\right)_{Mass}=\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)dxdy$$

$$\left(\dot{E}_{out}-\dot{E}_{in}\right)_{Heat}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)dxdy$$

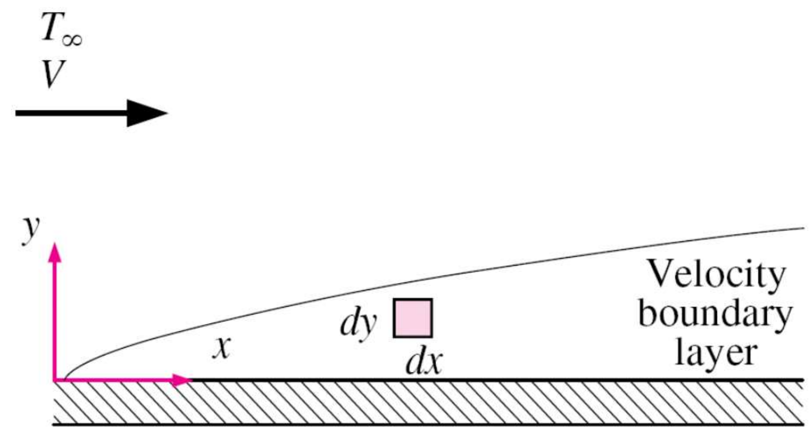


Therefore, Eq. (3.13) can be simplified to:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad \ldots\ldots (3.14)$$

4.3.1.5 Governing Equations for forced convection

❑ Energy Equation



Assumptions

- laminar flow over a flat plate
- steady two-dimensional flow
- Newtonian fluid with constant properties.
- viscous effects are significant

Therefore, (3.13) simplifies to:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \Phi \ldots \ldots \text{(3.15)}$$

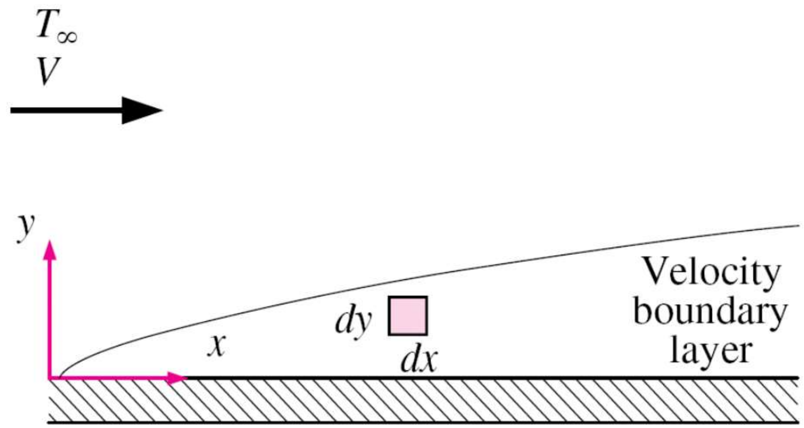
Where,

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2$$

4.3.1.5 Governing Equations for forced convection

Assumptions

- laminar flow over a flat plate
- steady two-dimensional flow
- Newtonian fluid with constant properties.
- viscous effects are significant



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots\dots (3.11) \qquad \text{Continuity}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \dots\dots (3.12) \qquad \text{Momentum}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi \qquad \dots\dots (3.15) \qquad \text{Energy}$$

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$