ME265: Thermal Engineering & Heat Transfer

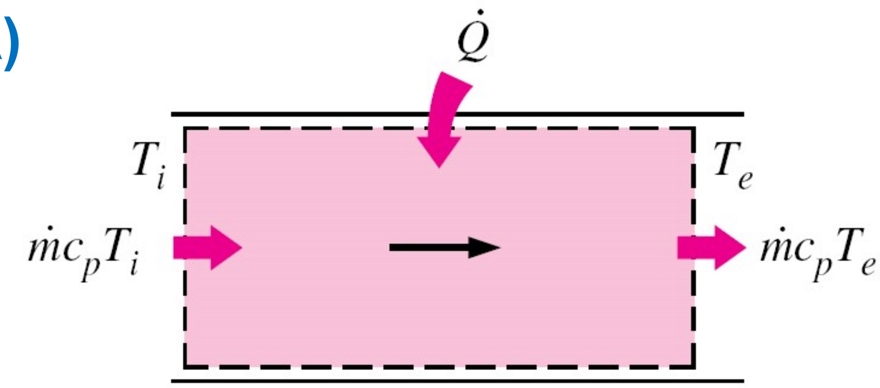
Chapters
1. Energy Scenario
2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

4.1 Introduction	
4.2 Conduction	
4.3 **Convection**	**4.3.1 Convection Fundamentals** **4.3.2 External Forced Convection** **4.3.3 Internal Forced Convection** **4.3.4 Natural Convection**
4.4 Radiation	
4.5 Heat Exchanger	

4.3.3 Internal Forced Convection

❑ General Thermal Analysis (GTA)

In the absence of any work interactions, the conservation of energy equation for the steady flow in a tube is given by:



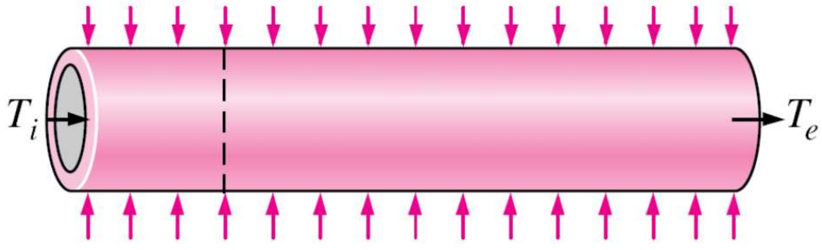
$$\dot{Q} = \dot{m}c_p\left(T_e - T_i\right) \quad \text{… … (3.3.3)}$$

Where, T_i and T_e are the mean fluid temperatures at inlet and exit

- The thermal conditions at the surface can usually be approximated as:
  - ➢ **constant surface temperature**
  - ➢ **constant surface heat flux**.

❑ GTA for Constant surface heat flux

- The rate of heat transfer can be expressed as



$$\dot{Q} = \dot{q}_s A_s$$

$$\dot{Q} = \dot{m} c_p \left(T_e - T_i\right) \quad \text{… … (3.3.3)}$$

- Then the mean fluid temperature at the tube exit becomes

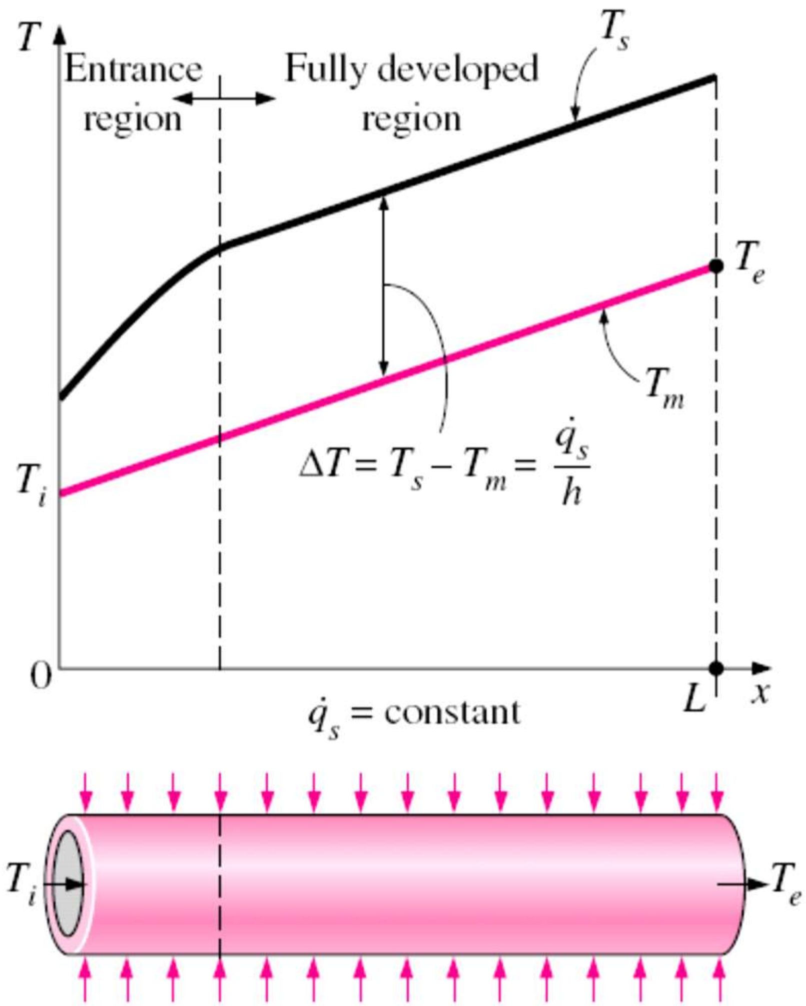
$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} c_p} \quad \text{… … (3.3.4)}$$

- Again, the surface temperature can be determined from

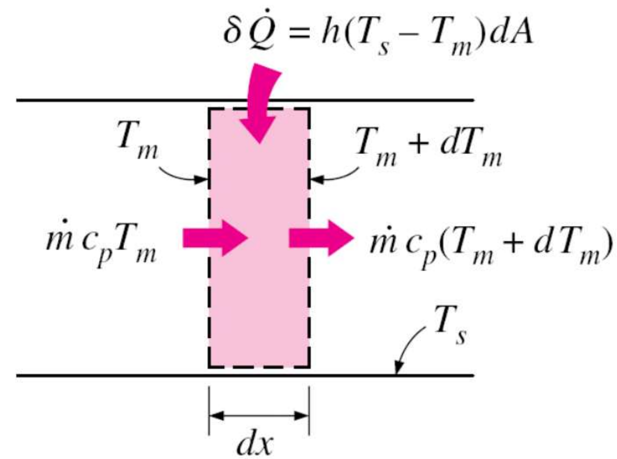
$$\dot{q}_s = h\left(T_s - T_m\right) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h} \quad \text{… … (3.3.5)}$$

$$\Rightarrow \quad \frac{dT_s}{dx} = \frac{dT_m}{dx}$$

❑ GTA for Constant surface heat flux



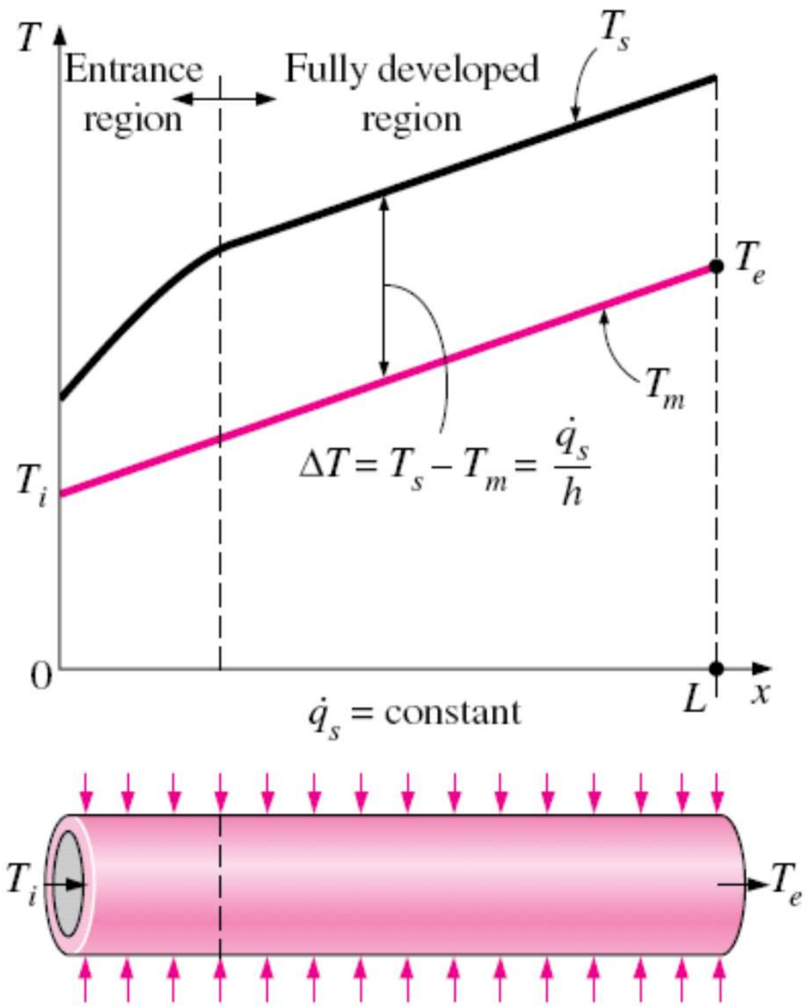
$$\frac{dT_s}{dx} = \frac{dT_m}{dx}$$



$$\dot{m}c_p dT_m = \dot{q}_s\left(p dx\right)$$

$$\rightarrow \quad \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant}$$

❑ GTA for Constant surface heat flux



$$\frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = constant$$

In the fully developed region:

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$

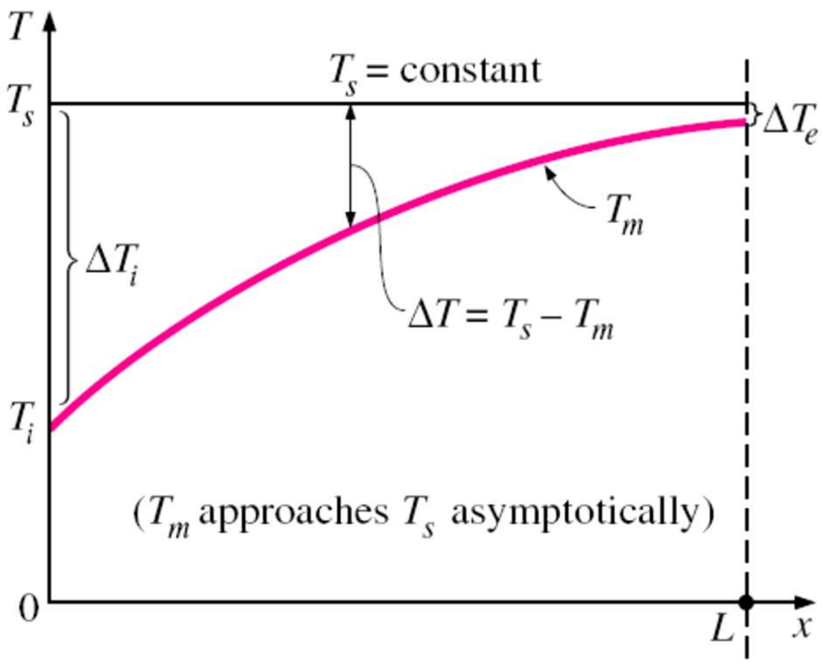
$$\rightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0$$

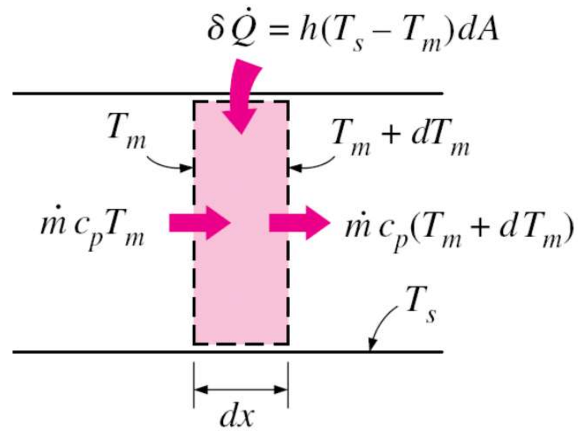
$$\rightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

Therefore,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = constant$$

❑ GTA for Constant surface temperature





Energy Balance:

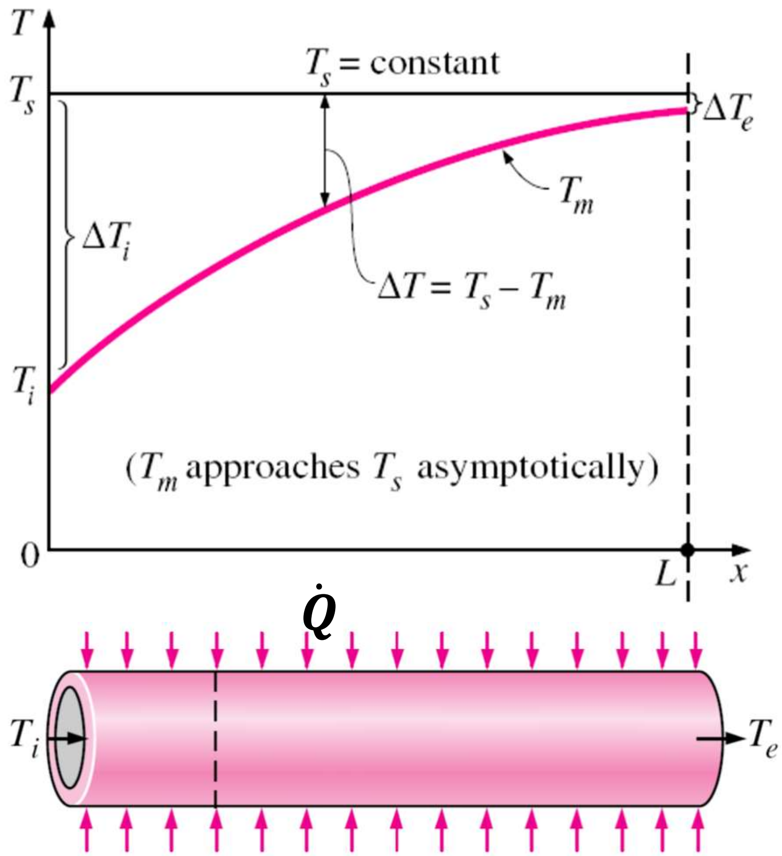
$$\delta \dot{Q} = \dot{m} c_p dT_m = h\left(T_s - T_m\right) dA_s$$

$$\Rightarrow \frac{d(T_s - T_m)}{(T_s - T_m)} = \frac{h\, dA_s}{\dot{m}\, c_p}$$

- Integrating from x=0 (at inlet where T_m=T_i) to x=L (at exit where T_m=T_e) gives

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} c_p} \qquad \text{… … (3.3.6)}$$

❑ GTA for Constant surface temperature



$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p} \quad \text{… … (3.3.6)}$$

$$\Rightarrow \dot{m}c_P = -\frac{hA_s}{ln[(T_s - T_e)/(T_s - T_i)]}$$

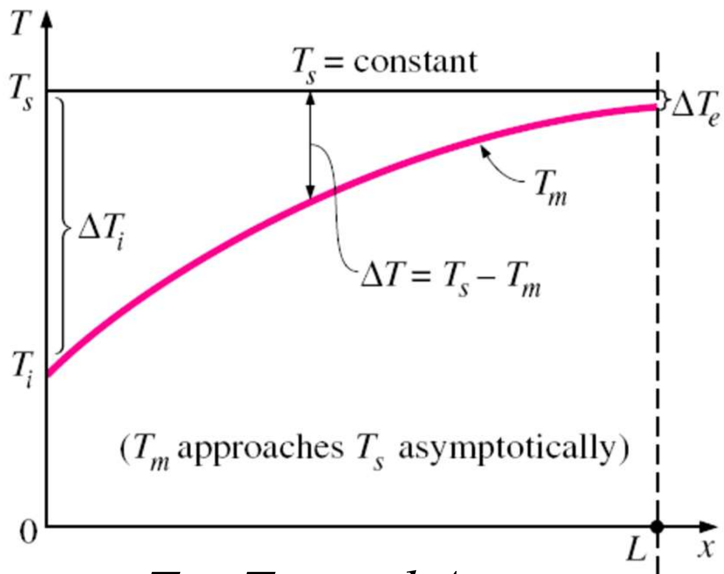
$$\dot{Q} = \dot{m}c_P(T_e - T_i)$$
$$= h\,A_s\,\Delta T_{lm}$$

Here, ΔT_{lm} is the logarithmic mean temperature difference given by:

$$\Delta T_{lm} = \frac{T_i - T_e}{ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{ln[\Delta T_e/\Delta T_i]} \quad \text{… … (3.3.7)}$$

4.3.3 Internal Forced Convection: GTA

❑ Constant T_s



$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p}$$

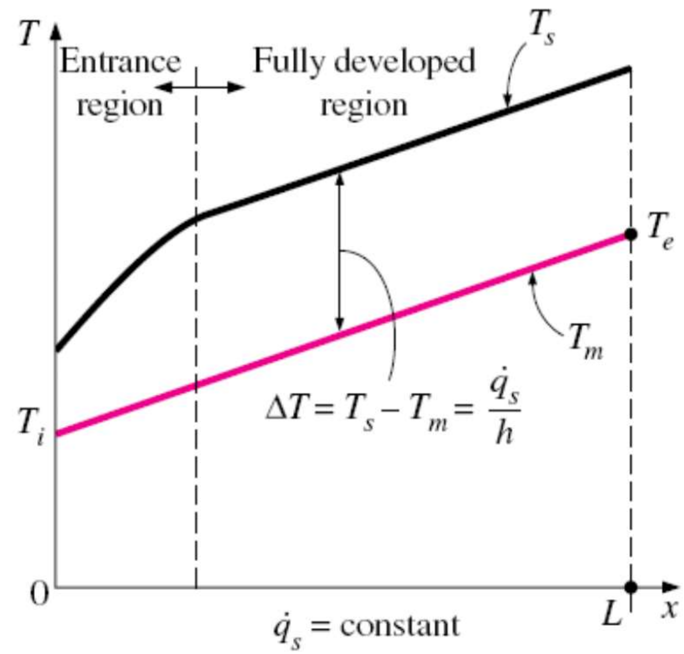
$$\Rightarrow T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right)$$

$$\dot{Q} = h\, A_s\, \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{T_i - T_e}{ln[(T_s - T_e)/(T_s - T_i)]}$$

$$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{ln[\Delta T_e/\Delta T_i]}$$

❑ Constant $\dot{q}_s$



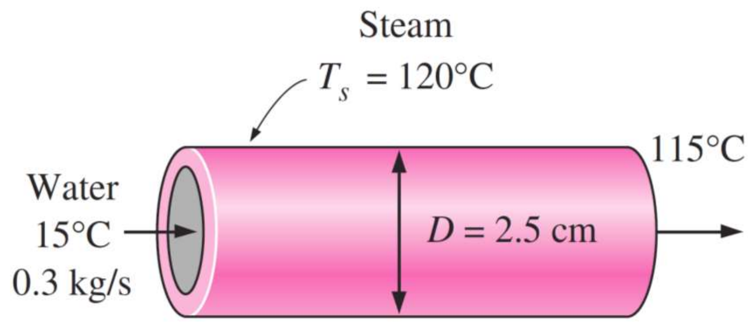
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p}$$

$$\dot{Q} = \dot{m}c_P(T_e - T_i)$$

$$\dot{Q} = h(T_s - T_m)$$

EP# 3.8 Cengel et al. Example: 8-3

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m^{2}C, determine the length of the tube required in order to heat the water to 115°C.



Assumptions:

1. Steady operating conditions exist.
2. Fluid properties are constant.
3. The convection heat transfer coefficient is constant.
4. The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties

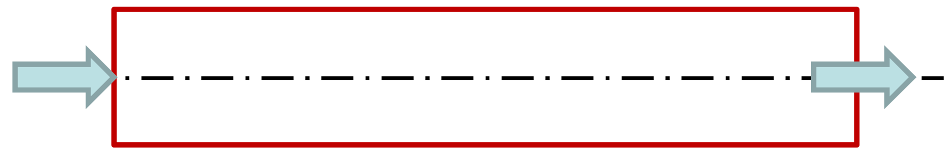
c_p=4187 J/kg°C for water at the **bulk mean temperature** of (15 + 115)/2 = 65°C

h_{fg}=2203 kJ/kg for steam at 120°C(Table A-9).

4.3.3 Internal Forced Convection

❑ Laminar Forced Convection in a Pipe

- Velocity profile,
- Pressure drop,
- Temperature profile,
- Convection coefficient, Nu

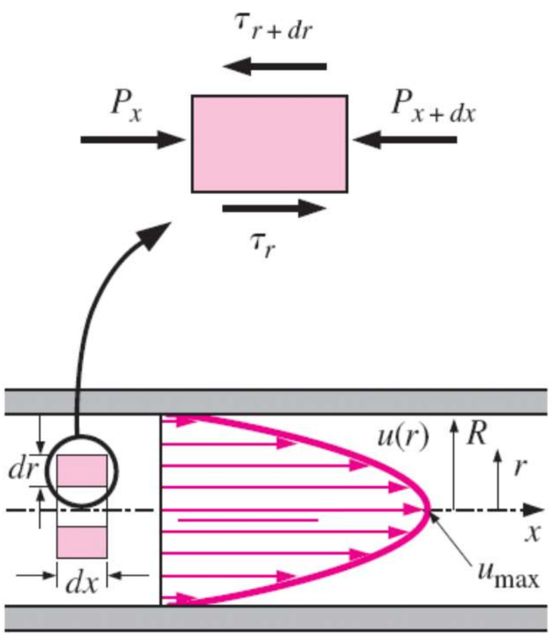


Assumptions:

- steady laminar flow,
- incompressible fluid,
- constant properties,
- fully developed region, and
- straight circular pipe.

- The velocity profile ***u*(*r*)** remains unchanged in the flow direction.
- no motion in the radial direction=> $v=0$.

❑ Velocity Profile during LFC in a Pipe



- Force balance in the flow direction gives:

$$\sum_x F = 0$$

$$\Rightarrow (2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dr \tau)_r - (2\pi r dr \tau)_{r+dr} = 0$$

$$\Rightarrow r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$\Rightarrow r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

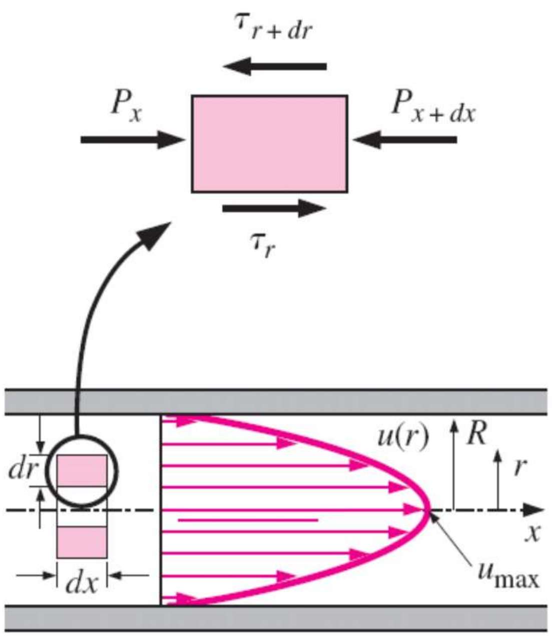
Substituting $\tau = \mu \frac{du}{dr}$ $\Rightarrow$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Rearranging and integrating $\Rightarrow$

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \quad \text{…… (3.3.8)}$$

❑ Velocity Profile during LFC in a Pipe



$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \quad \text{… … (3.3.8)}$$

- **Boundary conditions:**
  - For min/max velocity:

    $\frac{du}{dr} = 0$ at $r = 0$,
  - No-slip condition, u=0 at r=R

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \quad \text{… … (3.3.9)}$$

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)\,r\,dr = -\frac{2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr$$

$$= -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right) \quad \text{… … (3.3.10)}$$

At r=0, $u_{max} = 2V_{avg}$

❑ Temperature Profile during LFC in a Pipe

- The steady flow energy balance for a cylindrical shell element can be expressed as

$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

- Substituting $\dot{m} = \rho u A_c = \rho u(2\pi r dr)$ and rearranging, we get:

$$\rho c_p u \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

$$\Rightarrow \quad u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r dx}\frac{\partial \dot{Q}}{\partial r}$$

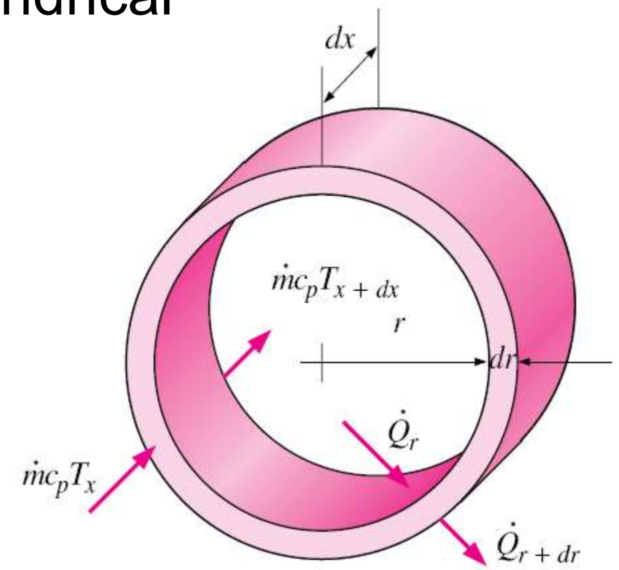


Fig. Cylindrical shell element

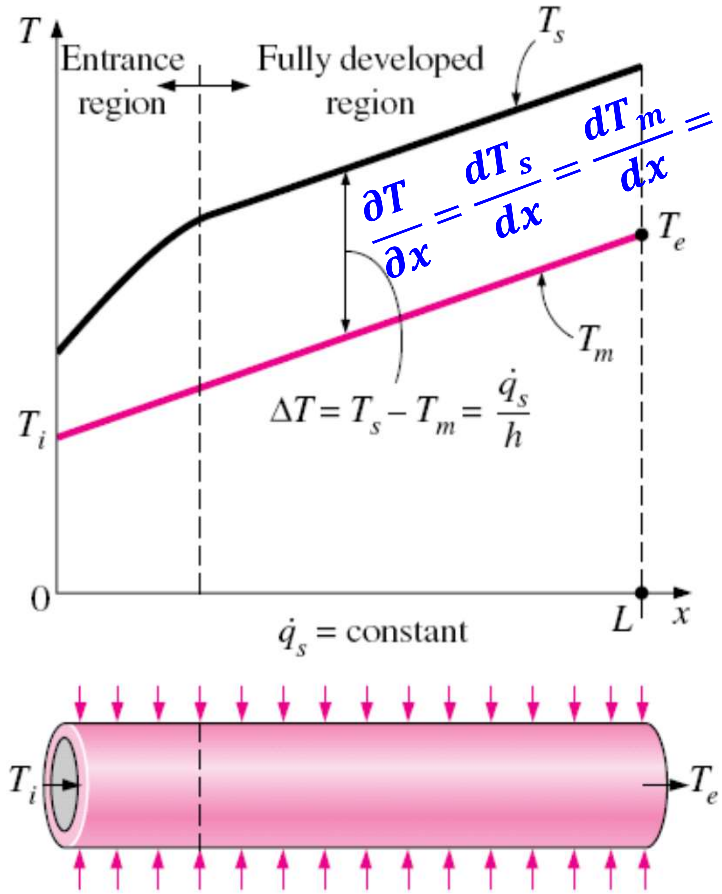
- Using Fourier Law of Conduction, $\dot{Q} = -k\,(2\pi r dx)\frac{\partial T}{\partial r}$, we get:

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad \dots\dots (3.3.11)$$

Where, $\alpha = \frac{k}{\rho c_p}$

4.3.3 Internal Forced Convection: LFC

❑ Temperature Profile for Constant heat flux



$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = \frac{2\dot{q}_s}{\rho V_{avg} c_p R}$$

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad \dots\dots (3.3.11)$$

Substituting u and $\partial T/\partial x$

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

- **Boundary conditions:**
  - For min/max temp, $\frac{dT}{dr} = 0$ at $r = 0$,
  - At r=R, T=T_s

- **Applying Boundary conditions, we get:**

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \quad \dots\dots (3.3.12)$$

4.3.3 Internal Forced Convection: LFC

❑ Nusselt Number for Constant heat flux

$$Nu = \frac{hD}{k} = \frac{\dot{q}_s D}{k(T_s - T_m)} \quad \ldots \ldots (3.3.13)$$

Using $\dot{q}_s = h(T_s - T_m)$

Where,

$$T_m = \frac{2}{V_{avg} R^2} \int_0^R T(r) u(r) r dr$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

By integration,

$$T_s - T_m = \frac{11}{24}\frac{\dot{q}_s R}{k}$$

Using (3.3.13), we get: $Nu = \frac{48}{11} = 4.36 \quad \ldots \ldots (3.3.14)$

❑ Nu for Constant Surface Temperature

$$Nu = 3.66 \quad \ldots \ldots (3.3.15)$$