ME265: Thermal Engineering & Heat Transfer

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2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

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4.2.4 Transient Heat Conduction

Heat conduction in solids with

- constant properties
- no heat generation
- heat flows in one direction

Coordinate System	Governing Equation (Transient)	Temperature distribution
Cartesian	$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	**T(x,t)**
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	**T(r,t)**
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	**T(r,t)**

Solutions methods:

- Exact solution by
  - -Separation of variables
  - -Laplace transform
  - -Similarity parameter
- Graphical methods

-Heisler Charts

α → Thermal diffusivity = k/ρc, m^2/s
k → Thermal conductivity, W/m.K

4.2.4 Transient Heat Conduction

❑ Lumped System Analysis

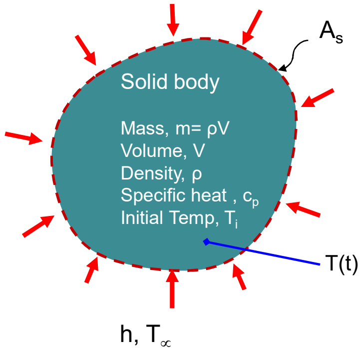
In heat transfer analysis, some materials behave like a **“lump”** whose interior **temperature remains** essentially **uniform** at all times.

Coordinate System	**For lumped system Temperature distribution**
Cartesian	**T(x,t) → T(t)**
Cylindrical	**T(r,t) → T(t)**
Spherical	**T(r,t) → T(t)**

Spatial variation of temperature is neglected
under certain conditions

4.2.4 Transient Heat Conduction

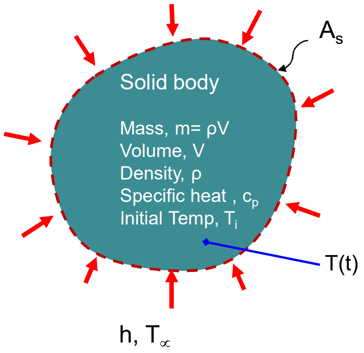
Lumped System Analysis



$$\left(\begin{array}{l}\textbf{Heat Transfer into}\\ \textbf{the body during } dt\end{array}\right) = \left(\begin{array}{c}\textbf{The increase in energy}\\ \textbf{of the body during } dt\end{array}\right)$$

4.2.4 Transient Heat Conduction

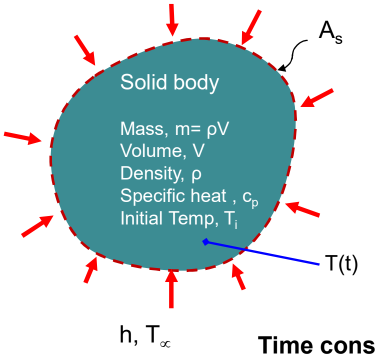
Lumped System Analysis

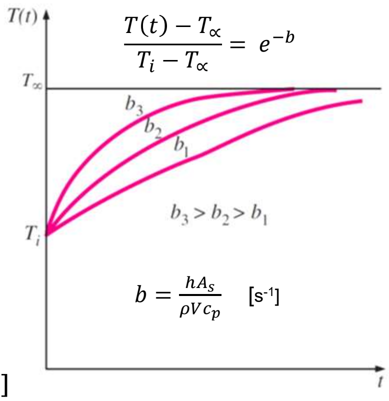


$$\begin{pmatrix}\textbf{Heat Transfer into}\\ \textbf{the body during } dt\end{pmatrix} = \begin{pmatrix}\textbf{The increase in energy}\\ \textbf{of the body during } dt\end{pmatrix}$$

4.2.4 Transient Heat Conduction

2.4.1 Lumped System Analysis





$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-b}$$

$$b = \frac{hA_s}{\rho V c_p} \quad [\mathrm{s}^{-1}]$$

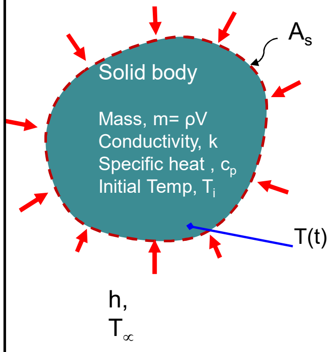
Time constant, $\boldsymbol{\tau = \frac{1}{b}\,[s]}$

❑ For faster response of temperature sensor,

- Time constant, τ should be smaller ➔ *b* should be larger
- Surface area to volume ratio (A_s/V) should be higher
- Heat capacity should be lower
- Heat transfer coefficient to be as high as possible

4.2.4 Transient Heat Conduction

2.4.1 Lumped System Analysis: Criteria



- Assumption of lumped system is not always appropriate
- The first step in establishing a criterion for the applicability is to define:
  - **characteristic length, L_c** of the solid, and
  - **dimensionless number** called **Biot number, Bi**

$$L_c = \frac{V}{A_s} \qquad Bi = \frac{hL_c}{k}$$

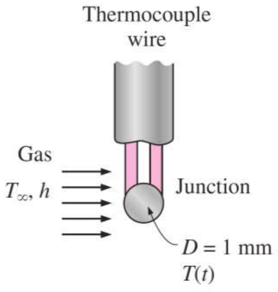
$$Bi = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection heat flux}}{\text{Conduction heat flux}}$$

$$= \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

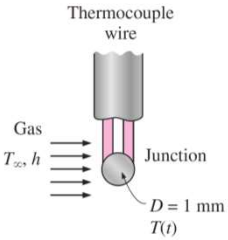
- ☐ For lumped analysis, **Bi** should be as small as possible
- ☐ Generally accepted condition: **Bi < 0.1**

EP#2.7 Thermocouple response time (Cengel et. al. Example 4-1)

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere. The properties of the junction are k = 35 W/m°C, ρ=8500 kg/m^3, and c_p = 320 J/kg°C, and the convection heat transfer coefficient between the junction and the gas is h = 210 W/m^2°C. **Determine how long it will take for the thermocouple to read 99% of the initial temperature difference**.



EP#2.7--Solution



$$b = \frac{hA_s}{\rho V c_p}$$

$$\frac{T(t) - T_\propto}{T_i - T_\propto} = e^{-bt}$$

EP#2.7--Solution