ME265: Thermal Engineering & Heat Transfer

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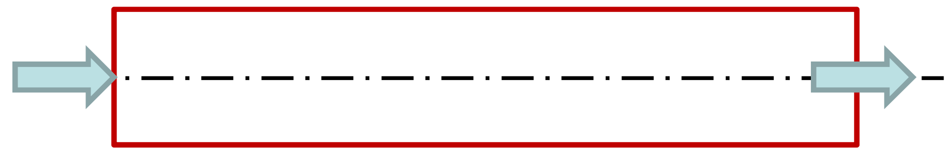
4.3.3 Internal Forced Convection

❑ Laminar Forced Convection in a Pipe

- Velocity profile,
- Pressure drop,
- Temperature profile,
- Convection coefficient, Nu

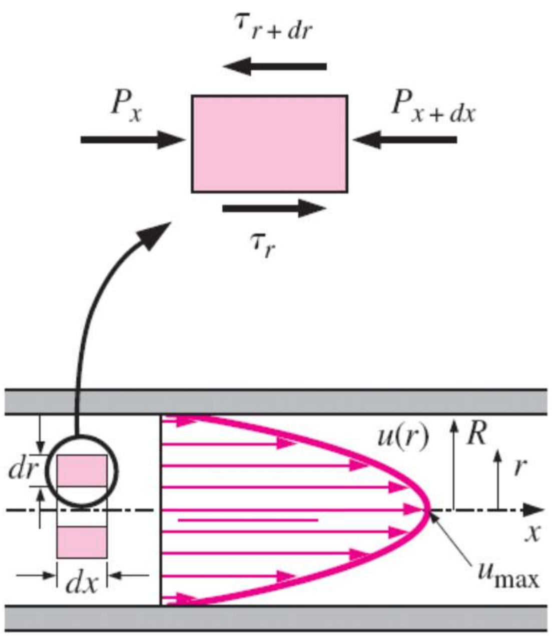


Assumptions:

- steady laminar flow,
- incompressible fluid,
- constant properties,
- fully developed region, and
- straight circular pipe.

- The velocity profile ***u*(*r*)** remains unchanged in the flow direction.
- no motion in the radial direction=> v=0.

❑ Velocity Profile during LFC in a Pipe



- Force balance in the flow direction gives:

$$\sum_x F = 0$$

$$\Rightarrow \left(2\pi r dr P\right)_x - \left(2\pi r dr P\right)_{x+dx} + \left(2\pi r dr \tau\right)_r - \left(2\pi r dr \tau\right)_{r+dr} = 0$$

$$\Rightarrow r\frac{P_{x+dx} - P_x}{dx} + \frac{\left(r\tau\right)_{r+dr} - \left(r\tau\right)_r}{dr} = 0$$

$$\Rightarrow r\frac{dP}{dx} + \frac{d\left(r\tau\right)}{dr} = 0$$

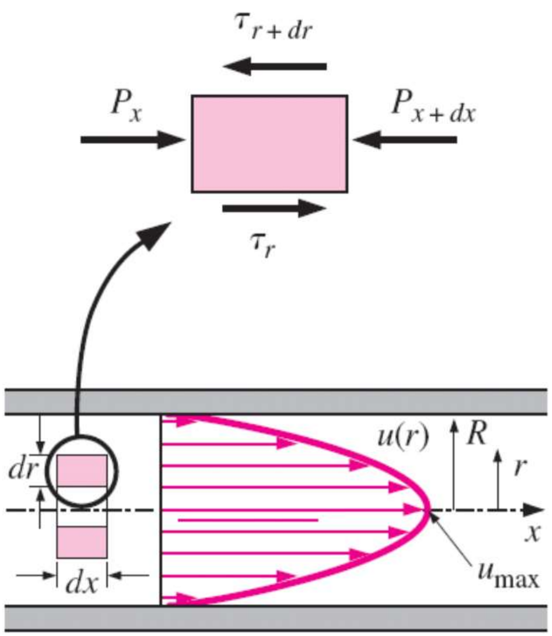
Substituting $\tau = \mu \frac{du}{dr}$

$$\Rightarrow \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Rearranging and integrating

$$\Rightarrow u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \quad \text{…… (3.3.8)}$$

❑ Velocity Profile during LFC in a Pipe



$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \quad \text{…… (3.3.8)}$$

- **Boundary conditions:**
  - For min/max velocity:

    $\frac{du}{dr} = 0$ at $r = 0$,
  - No-slip condition, u=0 at r=R

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \quad \text{…… (3.3.9)}$$

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)\,r\,dr = -\frac{2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr$$

$$= -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right) \quad \text{…… (3.3.10)}$$

At r=0, $$u_{max} = 2V_{avg}$$

❑ Temperature Profile during LFC in a Pipe

- The steady flow energy balance for a cylindrical shell element can be expressed as

$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

- Substituting $\dot{m} = \rho u A_c = \rho u(2\pi r dr)$ and rearranging, we get:

$$\rho c_p u \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

$$\Rightarrow \quad u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r dx}\frac{\partial \dot{Q}}{\partial r}$$

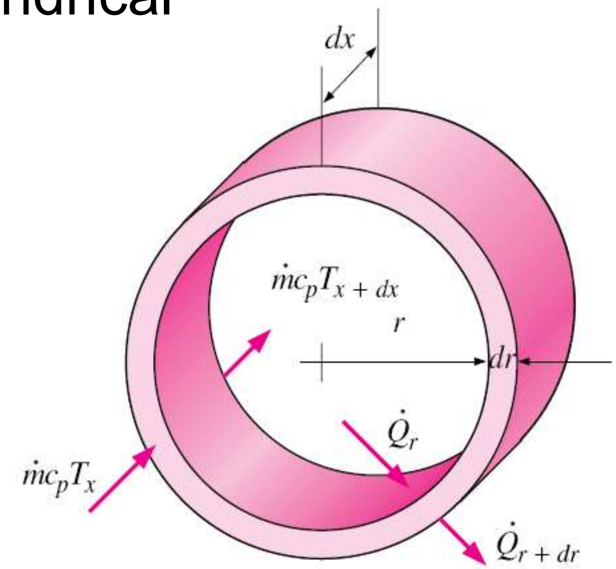


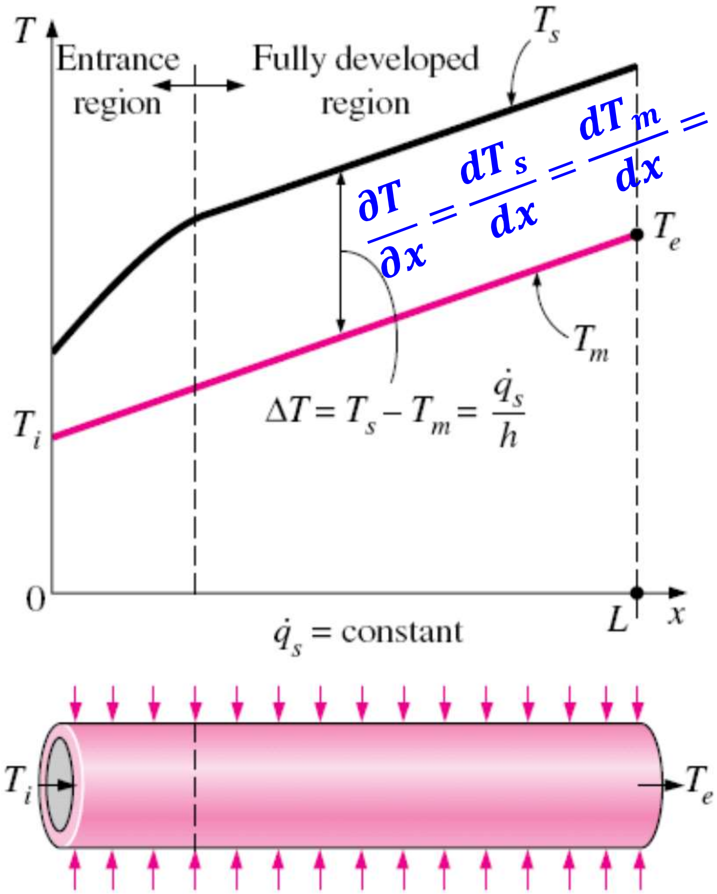
Fig. Cylindrical shell element

- Using Fourier Law of Conduction, $\dot{Q} = -k\,(2\pi r dx)\frac{\partial T}{\partial r}$, we get:

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad \dots \dots \text{(3.3.11)}$$

Where, $\alpha = \dfrac{k}{\rho c_p}$

❑ Temperature Profile for Constant heat flux



$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = \frac{2\dot{q}_s}{\rho V_{avg} c_p R}$$

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad \text{…… (3.3.11)}$$

Substituting u and $\partial T/\partial x$

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

- **Boundary conditions:**
  - For min/max temp, $\frac{dT}{dr} = 0$ at $r = 0$,
  - At r=R, T=T_s

- **Applying Boundary conditions, we get:**

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \quad \text{…… (3.3.12)}$$

❑ Nusselt Number for Constant heat flux

$$Nu = \frac{hD}{k} = \frac{\dot{q}_s D}{k(T_s - T_m)} \quad \ldots\ldots (3.3.13)$$

Using $\dot{q}_s = h(T_s - T_m)$

Where,

$$T_m = \frac{2}{V_{avg} R^2} \int_0^R T(r) u(r) r dr$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

By integration,

$$T_s - T_m = \frac{11}{24}\frac{\dot{q}_s R}{k}$$

Using (3.3.13), we get: $Nu = \frac{48}{11} = 4.36$ ……(3.3.14)

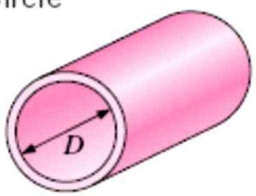
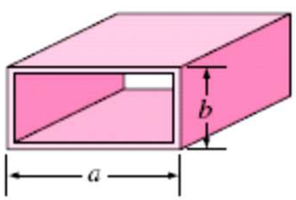
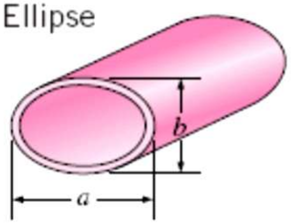
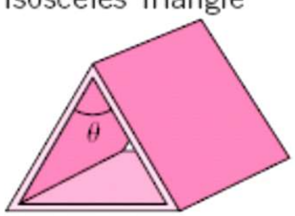
❑ Nu for Constant Surface Temperature

$$Nu = 3.66 \quad \ldots\ldots (3.3.15)$$

4.3.3 Internal Forced Convection: LFC

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = V_{avg}D_h/\nu$, and $\text{Nu} = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

$$L_{h,\text{laminar}} \approx 0.05\,\text{Re}\cdot D$$

$$L_{t,\text{laminar}} \approx 0.05\,\text{Re}\cdot \text{Pr}\cdot D$$

$$= \text{Pr}\cdot L_{h,\text{laminar}}$$

$$L_{h,turbulent} \approx L_{t,turbulent} \approx 10D$$

Laminar flow : Re < 2300

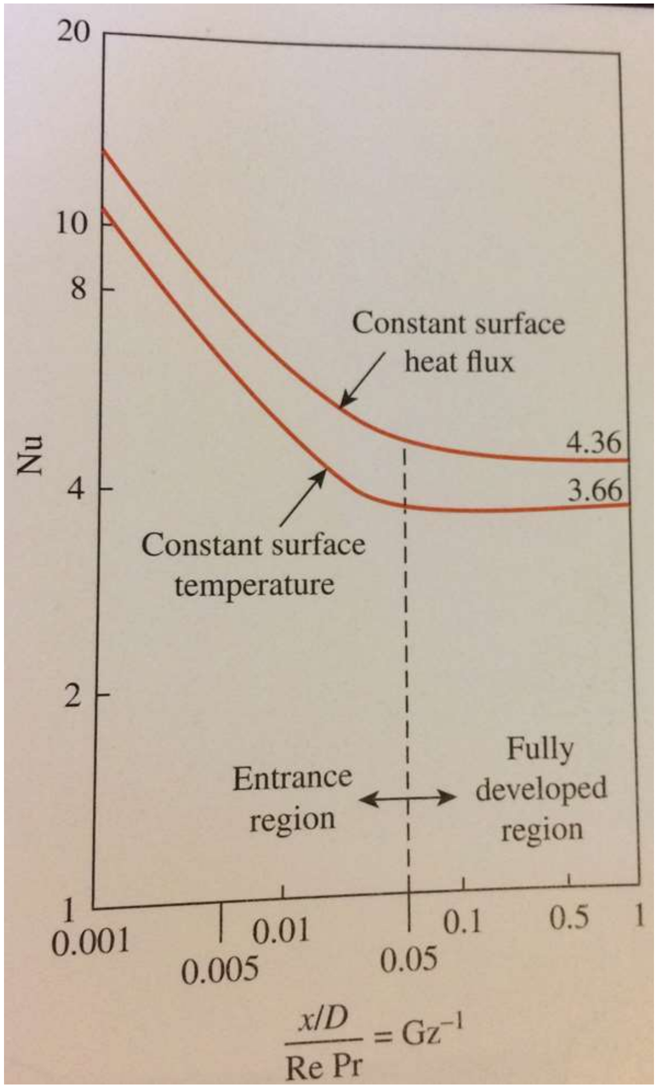
For a circular tube (D, L)

- Hydrodynamically developed and thermally developing flow **(Edwards et al. 1979):**

$$Nu = 3.66 + \frac{0.065\left(D/L\right)\mathrm{Re}\cdot\mathrm{Pr}}{1+0.04\left[\left(D/L\right)\mathrm{Re}\cdot\mathrm{Pr}\right]^{2/3}}$$

- Both hydrodynamically and thermally developing flow **(Seider and Tate, 1936):**

$$Nu = 1.86\left(\frac{Re\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$



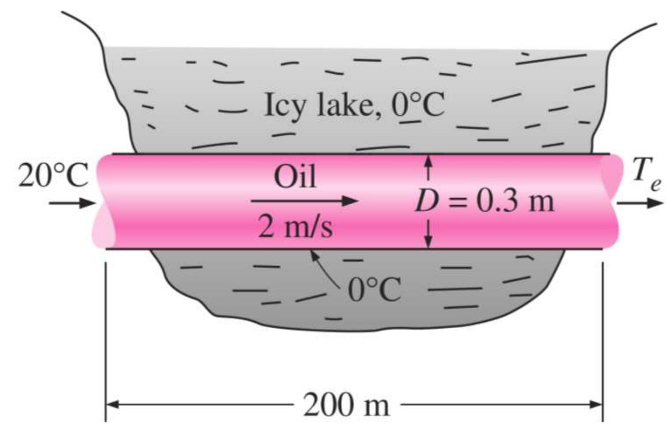
The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$, except for μ_s which is evaluated at surface temperature.

EP# 3.9

Consider the flow of oil at 20°C in a 30-cm-diameter pipeline at an average velocity of 2 m/s. A 200-m-long section of horizontal pipeline passes through a icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine—

(a) The temperature of the oil when the pipe leaves the lake

(b) The rate of heat transfer from the oil



Assumptions:

1. Steady operating conditions exist.
2. The surface temperature of the pipe is very nearly 0°C.
3. The thermal resistance of the pipe is negligible.
4. The inner surfaces of the pipeline are smooth.
5. The flow is hydrodynamically developed when the pipeline reaches the lake.

EP# 3.9 Soln

At 20°C we read (Table A-13)

ρ = 888 kg/m^3 ν = 9.429 × 10^{-4} m^2/s

k = 0.145 W/m°C c_p = 1881 J/kg °C

Pr = 10,860

$$Nu = 3.66 + \frac{0.065\left(D/L\right)\mathrm{Re}\cdot\mathrm{Pr}}{1+0.04\left[\left(D/L\right)\mathrm{Re}\cdot\mathrm{Pr}\right]^{2/3}}$$

4.3.3 IFC: Turbulent FC in pipes

- Most correlations for heat transfer coefficients in turbulent flow are based on experimental studies.
- For fully developed turbulent flow in *smooth tubes,* Nusselt number is calculated by the *Dittus–Boelter equation*

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \quad \begin{cases} \mathrm{Re} > 10,000 \\ 0.7 \le \mathrm{Pr} \le 160 \end{cases} \left| \begin{array}{l} n = 0.4 \; heating \\ n = 0.3 \; cooling \end{array} \right\}$$

- The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$.
- These are used when the difference between the fluid and the surface temperature is not large.

4.3.3 IFC: Turbulent FC in pipes

- When the difference between the fluid and the surface temperature is large, the Seider and Tate (1936) equation is used

$$Nu = 0.027\, Re^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \quad \begin{pmatrix} 0.7 \leq \mathrm{Pr} \leq 16{,}700 \\ Re \geq 10{,}000 \end{pmatrix}$$

- The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$, except for μ_s which is evaluated at surface temperature.

4.3.3 IFC: Turbulent FC in pipes

- The Nusselt number relations are fairly simple, but they may give errors as large as 25 percent.
- This error can be reduced considerably to less than 10 percent by using more complex but accurate relations such as the *second Petukhov equation:*

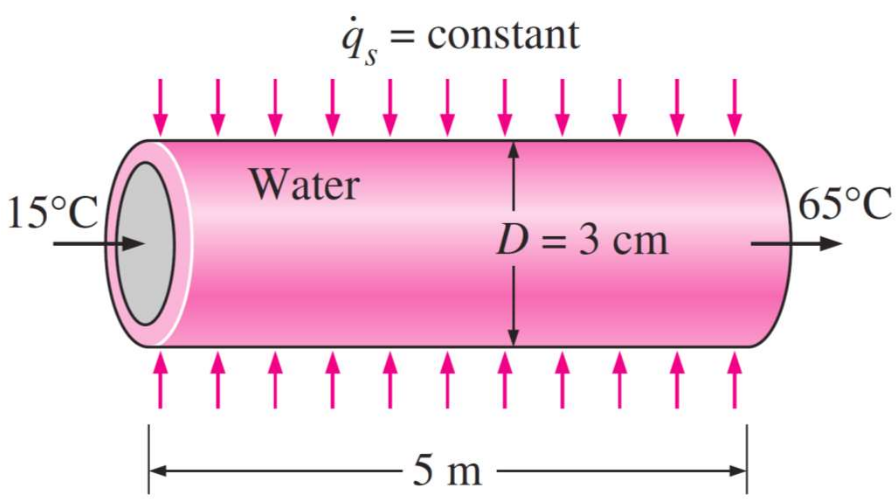
$$Nu = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7\,(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{Bmatrix} 0.5 \le Pr \le 2000 \\ 10^4 \le Re \le 5 \times 10^6 \end{Bmatrix}$$

$$f = (0.79 \ln \mathrm{Re} - 1.64)^{-2} \qquad 3000 < \mathrm{Re} < 5 \times 10^6$$

- The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$
- These are not very sensitive to thermal conditions at the tube surface

EP# 3.10

Water is to be heated from 15°C to 65°C as it flows through a 3-cm- internal diameter 5-m-long tube. The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.



Assumptions

1. Steady flow conditions exist.
2. The surface heat flux is uniform.
3. The inner surfaces of the tube are smooth.