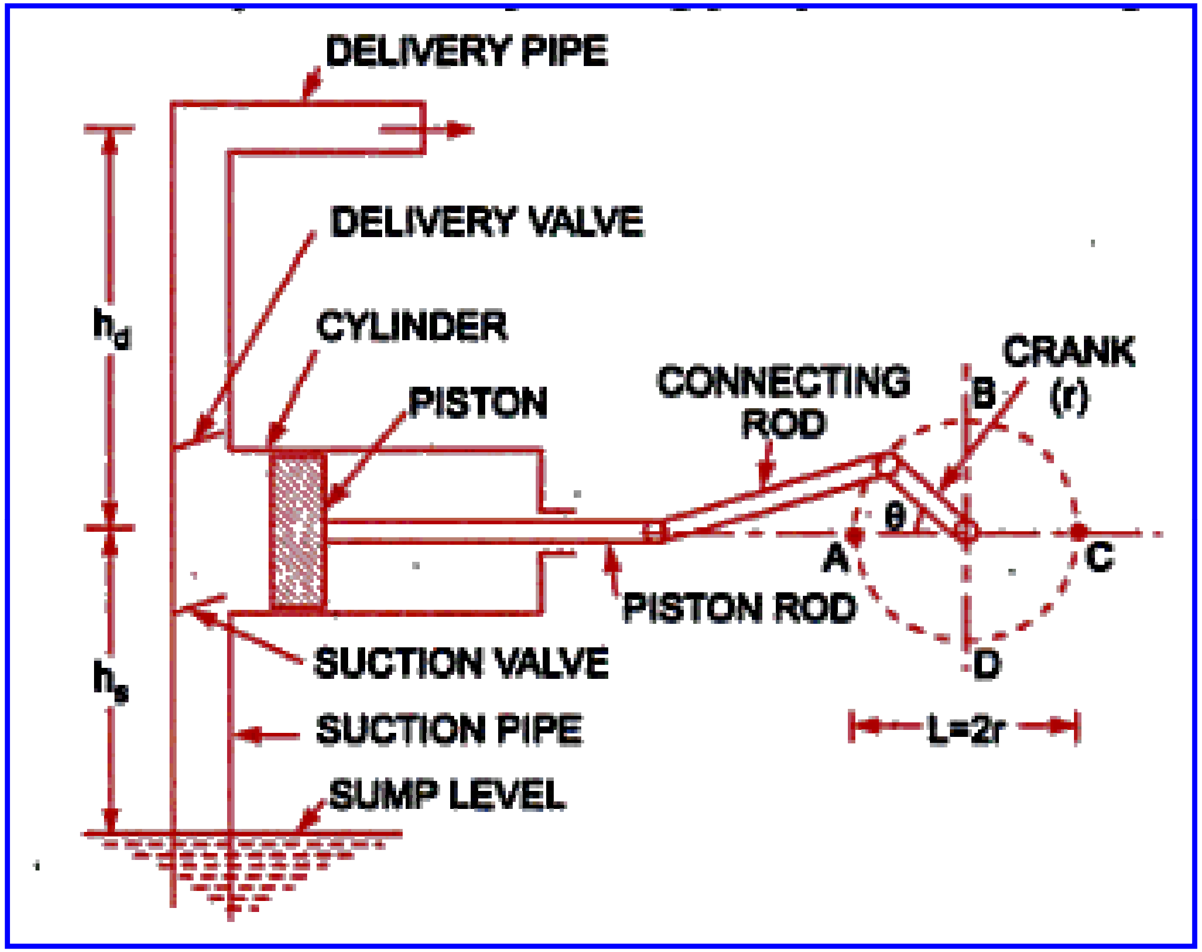
Reciprocating Pump



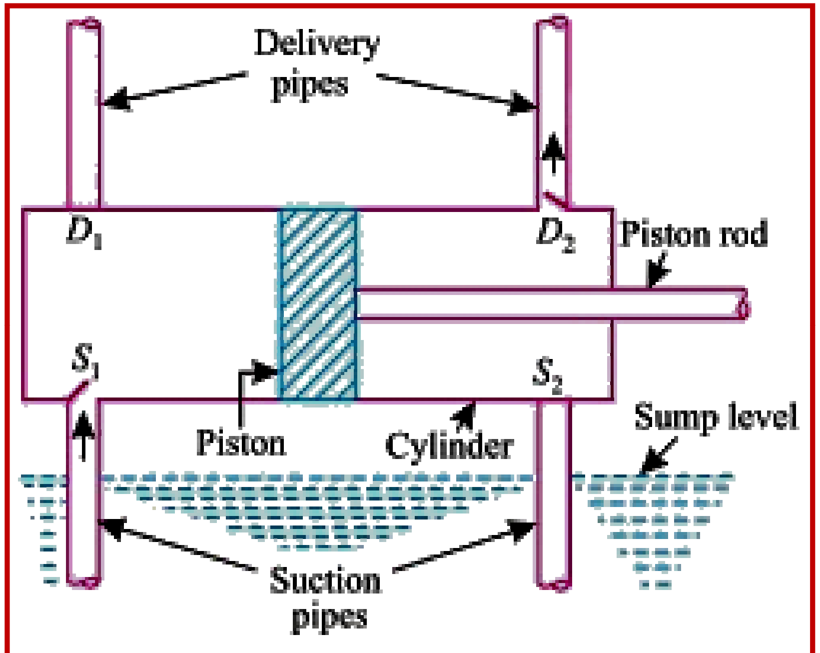
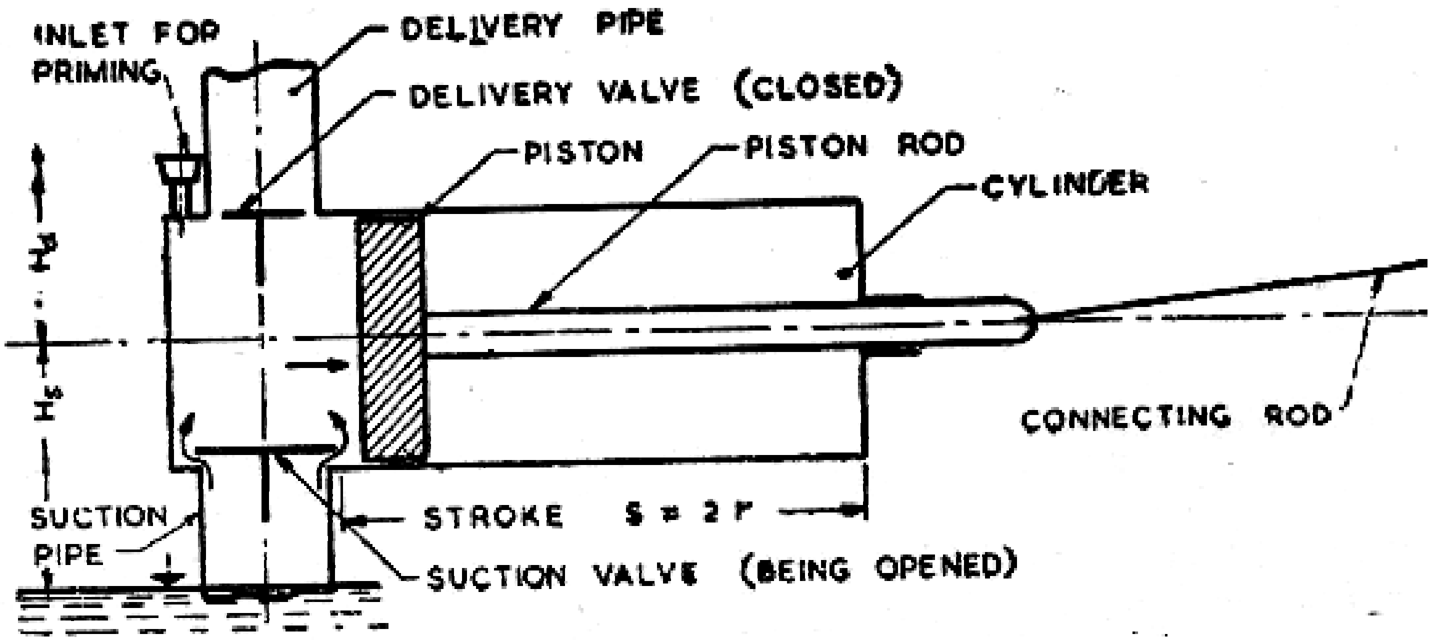


Fig. 4.2. Double-acting reciprocating pump.



A reciprocating pump ***consists primarily of a piston or a plunger reciprocating inside a close fitting cylinder***, **thus performing the suction and delivery strokes**. **The reciprocating pump is a positive acting type** which means ***it is a displacement pump which creates lift and pressure by displacing liquid with a moving member or piston.*** The chamber or cylinder is alternately filled and emptied by forcing and drawing the liquid by mechanical motion. This type is called "**positive**" inasmuch as the only limitation on pressure which may be developed is the strength of the structural parts. Suction and delivery pipes are connected to the cylinder as shown in Fig. ***Each of the two pipes is provided with a non-return valve***. The function of the non-return or one way valve is to ensure a unidirectional flow of liquid.

Thus the suction pipe valve allows the water only to enter the cylinder while the delivery pipe valve permits only its discharge from the cylinder. Volume or capacity delivered is constant regardless of pressure, and is varied only by speed changes.

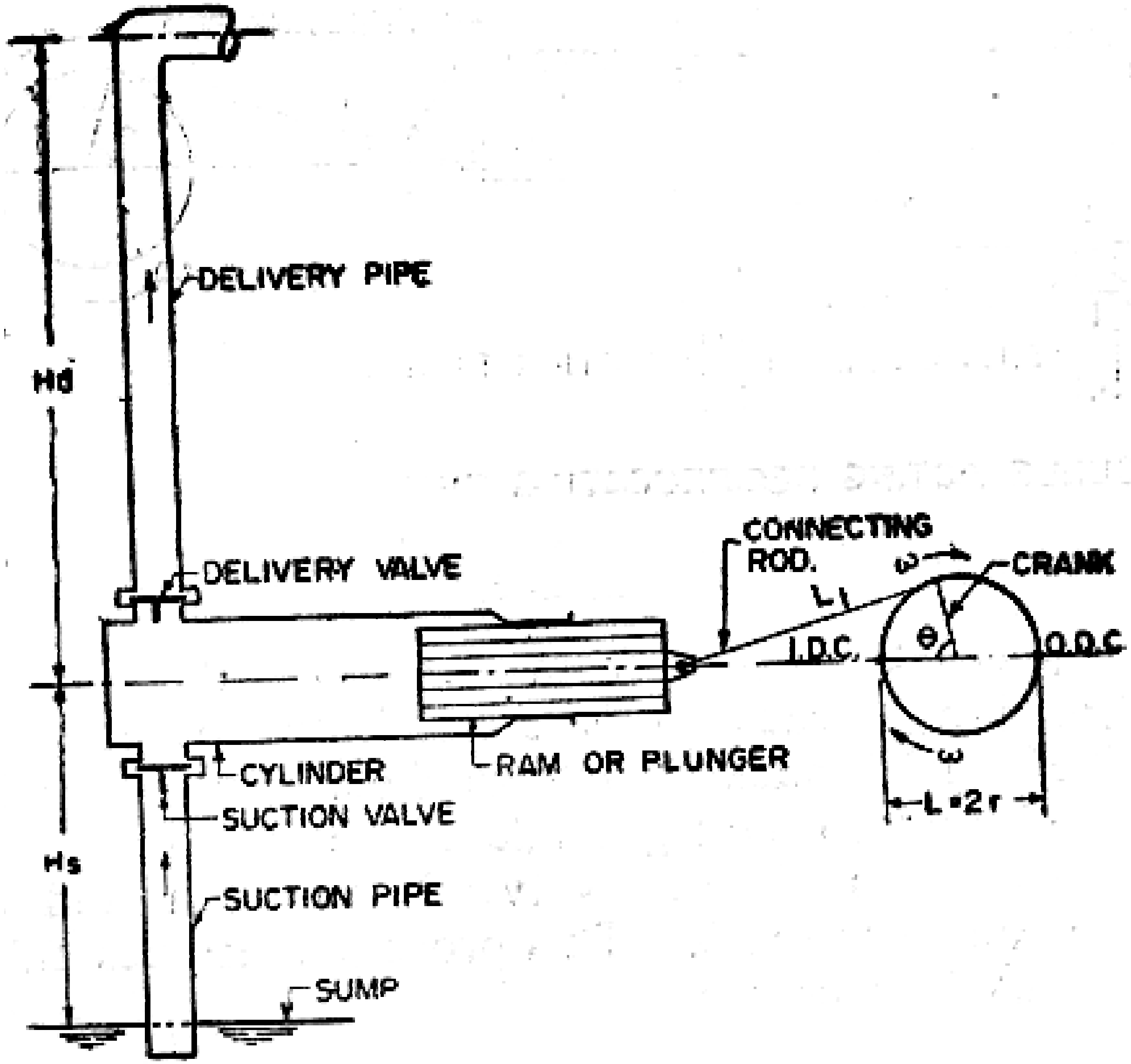
Applications

The reciprocating pump is ***best suited for relatively small capacities and high heads***. In ***oil drilling*** operations this type of pump is very common. The reciprocating pump is used generally for ***Pneumatic pressure systems***, ***feeding small boilers*** condensate return and light oil pumping.

Working of Single-acting Pump

In a single-acting reciprocating pump, **there is one suction pipe and one delivery pipe**. The pump is usually placed above the liquid level in the sump. When the crank rotates, the piston moves backward and forward in the cylinder. Let us suppose that initially the crank is at the inner-dead centre (I.D.C.). ***When the crank rotates in the clockwise direction, the piston moves towards right and a vacuum is created on the left side of the piston.***

The pressure acting on the liquid surface in the sump is equal to the ***atmospheric pressure***. Thus ***there is a pressure difference in the liquid*** at the two ends of suction *pipe* and ***the liquid is forced from the sump into the suction pipe***. The one-way valve in the suction pipe opens and the liquid fills the left side of the cylinder. At the end of the suction stroke, the crank is at the outer-dead centre (O.D.C.) and the cylinder is full of the liquid.



In the delivery stroke, ***the crank moves from the O.D.C. to the I.D.C. in the clockwise direction.***

This causes the piston to move from the right towards left. ***A high pressure is created in the cylinder.*** ***The delivery valve opens and the liquid is forced into the delivery pipe***. The liquid is carried to the reservoir through the delivery pipe. At the end of the delivery stroke, the crank comes to the I.D.C. and the piston is at the extreme left position. Thus one cycle is completed. As the crank rotates, the suction and delivery strokes are repeated.

Expression for Work done on a Reciprocating Pump

(i) In a single acting reciprocating Pump, in each revolution of the crank, there is one delivery stroke. If '*N*' is the speed of the crank, in r.p.m., the number of delivery strokes per minute is ***N***. Let 'L' be the length of the stroke and ***A*** be the cross-sectional area of the piston.

The **theoretical discharge** is given by $Q = ALN/60$ ------ (1)

Again, ***the length of stroke*** (***L***) ***is twice the length of crank*** (***r***).

Let 'H_s' is the height of the centre line of the cylinder above the liquid surface in the sump and

'H_d' is the height of the reservoir above the centre line of the cylinder

($-\gamma H_s$) is the vacuum pressure on the piston end during the suction stroke

(γH_d) the gauge pressure during the delivery stroke

($A\gamma H_s$) is the force on the piston during suction stroke.

($A\gamma H_d$) is the force on the piston during delivery stroke.

Work done during the suction stroke $= A\gamma H_s L$

Work done during the delivery stroke $= A\gamma H_d L$

Total work done in one cycle $= A\gamma L(H_s + H_d)$

Work done per second $= \dfrac{\gamma AL\,(H_s + H_d)N}{60}$ ------ (2)

From equs. (1) and (2), the work done per second may be written as

work done per second $= \gamma Q(H_s + H_d)$ ------ (3)

The **theoretical power** P_t, required to drive the pump is = $\gamma Q(H_s + H_d)$

------ (4)

The theoretical power (P_t) is also known as the **water power (W.P.)**.

Owing to the frictional and other losses, **the actual power required is more than that** given by equ. (4).

$$\text{Actual Power} = \frac{\text{Theoretical Power}}{\text{Efficiency of the pump}} = \frac{\gamma Q(H_s + H_d)}{\eta}$$ ------ (5)

The sum of the suction and delivery heads ***is known as the total static head.***

It is assumed that the discharge is equal to the volume of liquid swept by the piston. ***In practice***, it is found that the actual discharge (Q_a) is less than the theoretical discharge (Q). **The difference between the theoretical and the actual discharge is known** as **slip** *or* ***slip is the difference of volume swept through the piston and the actual discharge per stroke.***

Thus, **Slip** $= Q - Q_a$, and $$\text{Percentage slip} = \frac{Q - Q_a}{Q} \times 100 \qquad \text{------ (6)}$$

The slip is generally positive. But ***in some cases, when the delivery valve opens before the suction stroke is completed***, ***the actual discharge is more than the theoretical discharge***. **The slip in such cases is negative**.

The ratio of the actual discharge to the theoretical discharge is called the coefficient of discharge.

$$\text{Coefficient of discharge, } C_d = \frac{Q_a}{Q} = \frac{\text{actual discharge/stroke}}{\text{Volume swept/stroke}} \qquad \text{--- (7)}$$

Unless otherwise mentioned, the coefficient of discharge will be taken as unity. In other words, the slip is neglected. but **in case the slip is negative, C_d will be more than one.**

Problem:

Water is raised to a height of 20m above the sump level by a single acting reciprocating pump having a bore of 15cm and a stroke of 30 cm. If the pump has a speed of 60 r.p.m., **find the theoretical power and the theoretical discharge**. If the efficiency of the pump is 70%, **calculate the actual power**. If the pump has an actual discharge of 0.0052 cumecs, **find the percentage slip.**

Solution:

$$H_s + H_d = 20$$

$$\text{Work done per second} = \frac{\gamma AL\,(H_s + H_d)N}{60}$$

$$= \frac{9.81 \times \pi/4 \times 0.15^2 \times 0.3 \times 60 \times 20}{60} = 1.0396\ kN.m$$

Theoretical power = 1.0396 kW (1.41 h.p.).

Actual power $= \frac{1.0396}{0.7} = 1.485\ kW\ (2.01\ hp)$

Theoretical discharge = ALN/60 = $\pi/4 \times 0.15^2 \times 0.3 \times \frac{60}{60} = 0.0053\ m^3/s$

Percentage slip $= \frac{Q - Q_a}{Q} \times 100$

$$= \frac{0{\cdot}0053 - 0{\cdot}0052}{0{\cdot}0053} \times 100 = \frac{1}{53} \times 100 = 1{\cdot}88\%.$$

Problem:

A single acting reciprocating pump has its piston diameter as 15cm and stroke 25cm. The piston moves with simple harmonic motion and makes 50 double strokes per minute. The suction and delivery heads are 5m and 15m respectively. **Find, the force required to work the piston during the suction as well as delivery stroke**. Assume the efficiency of the suction and delivery strokes are as 60% and 75% respectively, **Determine the HP required, by the pump.**

Solution:

Given that, $D = 15cm$, $L = 25cm$, $H_s = 5m$, $H_d = 15m$,
$\eta_s = 60\% = 0.6$, $\eta_d = 75\% = 0.75$,
$N = 50$ double stroke per min $= 50$ rpm

The **force on the piston during suction stroke** $= A\gamma H_s / \eta_s$

$$= \frac{1{,}000 \times 5 \times \frac{\pi}{4} \times 0{\cdot}15^2}{0{\cdot}6}$$

= 147 kg or, 1.44 kN

The **force on the piston during delivery stroke** $= A\gamma H_d / \eta_d$

$$= \frac{1{,}000 \times 15 \times \frac{\pi}{4} \times 0{\cdot}5^2}{0{\cdot}75}$$

= 374 kg or, 3.67 kN

$$\text{HP required by the pump} = \frac{\text{total force} \times \text{distance moved per sec}}{75}$$

$$= \frac{\text{Force (suction} + \text{delivery)}}{75} \times \frac{S \, . \, N}{60}$$

$$= \frac{147 + 374}{75} \times 0{\cdot}25 \times \frac{50}{60}$$

$= \mathbf{1{\cdot}45\ HP}$ *Answer*

Indicator Diagram

The indicator diagram is a diagram which shows the pressure of liquid in the cylinder of the pump corresponding to any position of the piston during the suction and the delivery stroke. The ordinate of the diagram represents the pressure head and the abscissa represents the length of the stroke. ***The work done on the piston may be obtained directly from the indicator diagram***. Again, since **Volume = A × L** and A is constant, so **the volume is proportional to the length of the stroke.** Sometimes, the abscissa of the indicator diagram is used to represent the volume of the stroke.

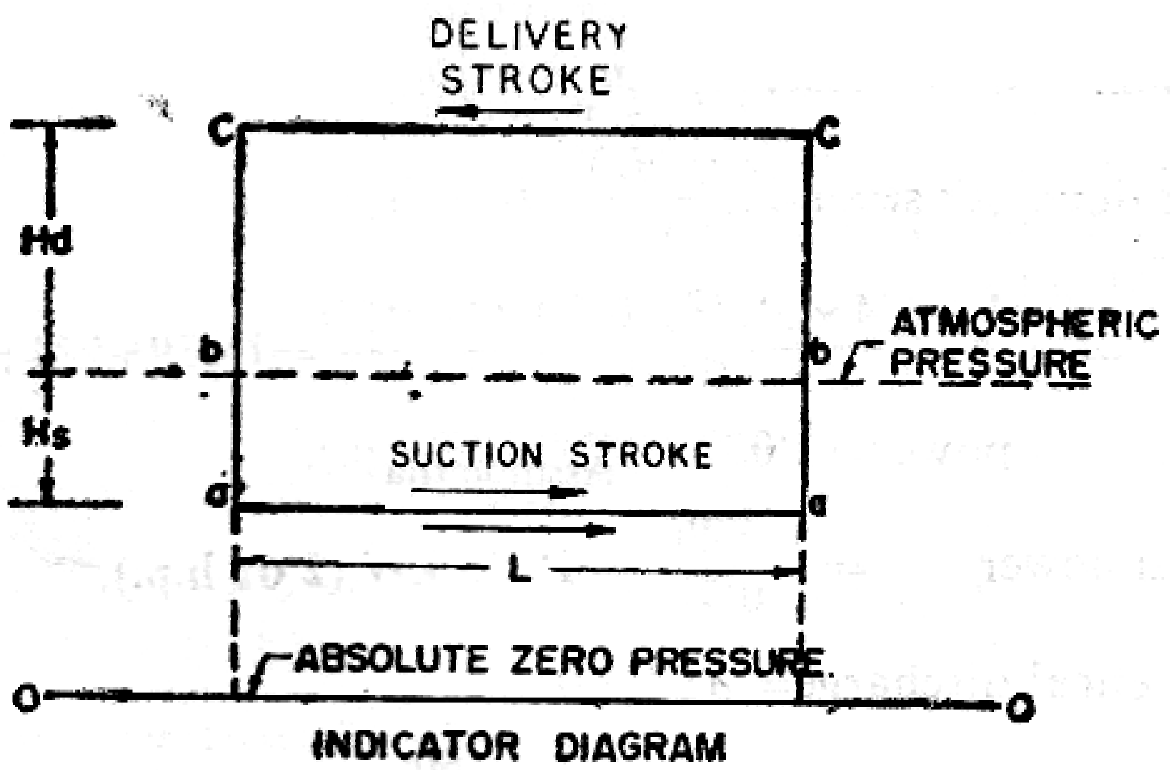


Fig. shows the **theoretical indicator diagram**. The line **O-O** shows the ***absolute zero pressure***. The line **b-b** represents the ***atmospheric pressure***. The pressure in the suction stroke being ***H_s (negative)***, it is represented by the line **a-a**.

The pressure in the delivery stroke is, **H_d (gauge)** and is represented by the line **c-c**.

The length of stroke is = L

The area of the indicator diagram is = $L(H_s + H_d)$

Evidently, **this is proportional to the work done**. Since from equ. (2)

Work done per second = $\gamma AN/60\ [L(H_s + H_d)]$, So

Work done per second = $\gamma AN/60$ × Area of indicator diagram.

Work done per revolution for single acting pump-

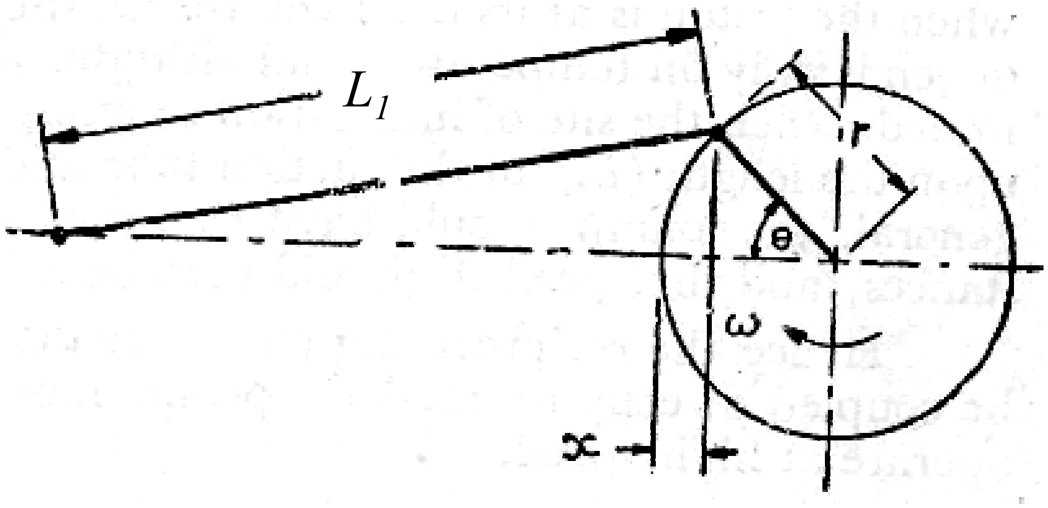
(i) during suction stroke = area **aabb**

(ii) during delivery stroke = area **bbcc**

The theoretical suction head is limited to 10.3 m of water if the atmospheric pressure acting on the sump is 101.043 kN/m^2 (1.03 kgf/cm^2). In practice. however, **because of separation of vapours, the suction head is limited to 7.70 m or so (for water)**. There is no such limitation on the delivery head.

Effect of Acceleration

The reciprocating motion of the piston causes an acceleration at the beginning of each stroke and a retardation at the end of each stroke. This transmits a corresponding acceleration and retardation to the water in the suction and delivery pipes. Because of inertia and the acceleration (or retardation), a force develops and there is variation of pressure in the cylinder.



If the length of connecting rod (L_1) is very long compared to the length of the crank, the motion of the piston is **simple harmonic**.

Consider, the position when the crank has moved through an angle **θ** from the inner dead centre (I.D.C). Let, ***ω*** be the **angular velocity** and ***t*** be the **time taken by the crank to rotate throughout the angle θ**. Therefore,

$$\theta = \omega t$$

Let x be the linear displacement of the piston from the beginning of the suction stroke. Therefore,

$$x = r - r\cos\theta = r - r\cos\omega t \qquad \text{-------(a)}$$

where r is the length of the crank (= L/2).

Velocity of the piston (V):

$$V = \frac{dx}{dt} = \frac{d}{dt}(r - r\cos\omega t) = \omega r \sin\omega t = \omega r \sin\theta \qquad \text{------- (b)}$$

Acceleration of the Piston:

$$F = \frac{dV}{dt} = \omega^2 r\cos\omega t = \omega^2 r\cos\theta \qquad \text{------- (c)}$$

Let v be the velocity of the liquid in the pipe and A be the area of the piston and a be the area of the pipe.

From the continuity consideration, since the volume of water flowing in the pipe equal to the volume of water flowing into the cylinder, so

$$AV = av \qquad or, \qquad v = AV/a$$

From equ. (b)

$$v = \frac{A}{a}(\omega r \sin\theta) \qquad \text{------- (d)}$$

The acceleration of water in the pipe is given by, $f = \frac{dv}{dt} = \frac{A}{a}(\omega^2 r \cos\theta) = \frac{A}{a}F$

Let ***l*** be the length of the pipe and **p_a** be the pressure due to acceleration of water. From Newton's 2nd Law of motion,

Force = Mass × Acceleration

$$p_a \times a = \frac{\gamma a l}{g} \times \left(\frac{A}{a}F\right)$$

$$p_a = \frac{\gamma l}{g} \times \frac{AF}{a} = \frac{\gamma l A}{a g}(\omega^2 r \cos\theta)$$

Pressure head due to acceleration , **$H_a = p_a/\gamma$**

$$H_a = \frac{lA}{ag}(\omega^2 r \cos\theta) \qquad \text{----- (8)}$$

Equ. (9) indicates the Pressure head due to acceleration (or retardation) of the piston varies with angle θ.

(i) At the **beginning of the stroke**, $\theta = 0$ and $H_a = \frac{lA}{ag}\omega^2 r$ ----- (9)

(ii) At the **middle of the stroke**, $\theta = 90°$ and $H_a = 0$ ----- (10)

(iii) At the end **of the stroke**, θ = 180° and $H_a = -\frac{lA}{ag}\omega^2 r$ ----- (11)

The above equations have been developed on the assumption that the motion is simple harmonic. **If the length of the connecting rod (L_1) is not very large as compared to the length of the crank (r)**, **the motion is far from simple harmonic**. For such a case, the acceleration of the piston is given by

At inner dead centre, $F = \omega^2 r\left[1 + \frac{r}{L_1}\right]$

At outer dead centre, $F = \omega^2 r\left[1 - \frac{r}{L_1}\right]$

Thus Eqs. (10) to (12) are modified as

For $\theta = 0$, $H_a = \frac{lA}{ag}\omega^2 r\left[1 + \frac{r}{L_1}\right]$ ----- (9a)

For $\theta = 90°$ $H_a = 0$ ----- (10a)

For $\theta + 180°$, $H_a = -\frac{lA}{ag}\omega^2 r\left[1 - \frac{r}{L_1}\right]$ ----- (11a)

(a) Effect of Acceleration in the Suction Pipe

Let us consider the piston at the beginning of the stroke. As the piston moves towards right, it should create not only a **negative pressure** equal to the suction head (**H_s**,) but it should also accelerate the water. Let the **acceleration head** be **H_{as}**. Thus the total negative pressure at the beginning of the suction stroke is (**H_s**+ **H_{as}**). To avoid separation, the absolute pressure at the beginning of the stroke should not fall below the vapour pressure. Let 'l_s' and 'a_s' be the length and the cross-sectional area of the suction pipe.

(i) From Eq. (10), the **at the beginning of the suction stroke,**

Acceleration head $$H_{as} = \frac{l_s \times A}{a_s \times g}\,\omega^2 r$$

Negative pressure, $$H_n = H_s + H_{as} = H_s + \frac{l_s \times A}{a_s \times g}\,\omega^2 r \quad \text{----- (12)}$$

Absolute pressure, $$H_s' = H_{atm} - H_n = H_{atm} - H_s - H_{as}$$

(ii) From Eq. (11), the **at the middle of the suction stroke,**

Acceleration head $H_{as} = 0$

Negative pressure, $H_n = H_s$ **----- (13)**

Absolute pressure, $H_s^{/} = H_{atm} - H_n = H_{atm} - H_s$

(iii) From Eq. (12), the **at the end of the suction stroke,** the water causes a positive pressure on the piston when it is retarding. This reduces the vacuum pressure in the cylinder.

Acceleration head $H_{as} = -\frac{l_s \times A}{a_s \times g}\,\omega^2 r$

Negative pressure, $H_n = H_s - H_{as} = H_s - \frac{l_s \times A}{a_s \times g}\,\omega^2 r$ **----- (14)**

The effect of acceleration head are:

1. No change in the work done.
2. Suction head is reduced.

This leads to the problem of separation in suction pipe in case the pressure at the beginning of suction is around 2.5 m of head of water (absolute). As the value depends on ω which is directly related to speed, the speed of operation of reciprocating pumps is limited.

(b) Effect of Acceleration in the Delivery Pipe

The water in the delivery pipe is accelerated at the beginning of the delivery stroke and retarded at the end of the delivery stroke, as in the case of suction pipe. Let 'l_d' and 'a_d' be the length and the cross-sectional area of the delivery pipe.

(i) at the **beginning of the delivery stroke**

Acceleration head $$H_{ad} = \frac{l_d A}{a_d g}\omega^2 r$$

Gauge pressure $$H_g = H_d + H_{ad} = H_d + \frac{l_d A}{a_d g}\omega^2 r \qquad \text{----- (15)}$$

Absolute pressure, $$H'_d = H_{atm} + H_g = H_{atm} + H_d + H_{ad} = H_{atm} + H_d + \frac{l_d A}{a_d g}\omega^2 r$$

(ii) at the **middle of the delivery stroke**

Acceleration head $$H_{ad} = 0$$

Gauge pressure $$H_g = H_d$$

Absolute pressure, $$H'_d = H_{atm} + H_g = H_{atm} + H_d \qquad \text{----- (16)}$$

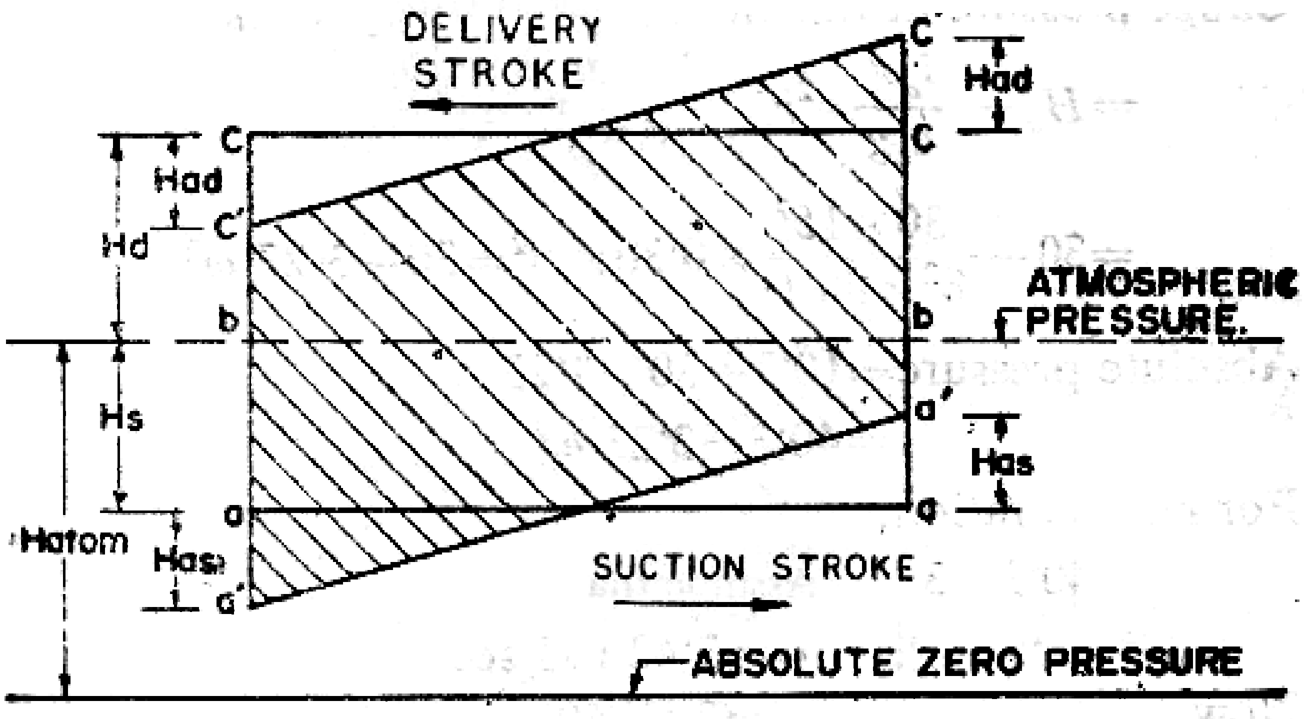
(ii) at the **end of the delivery stroke**

Acceleration head $$H_{ad} = -\frac{l_d A}{a_d g}\omega^2 r$$

Gauge pressure $$H_g = H_d + H_{ad} = H_d - \frac{l_d A}{a_d g}\omega^2 r \qquad \text{----- (17)}$$

Absolute pressure, $$H_d^{/} = H_{atm} + H_g = H_{atm} + H_d + H_{ad} = H_{atm} + H_d - \frac{l_d A}{a_d g}\omega^2 r$$

This absolute pressure should not be less than the vapour pressure to avoid separation.



EFFECT OF ACCELERATION HEAD

The indicator diagram modified for the acceleration head is shown in Fig. The **new diagram is represented** by ***a'a'c'c'***. It will be noted that ***the area of the indicator diagram remains unaffected. Thus the total work done remains the same***. **The main effect** of the acceleration head **is that it increases the negative head at the beginning of suction stroke**. If simple harmonic motion does not take place, the straight lines ***a'a'*** and ***c'c'*** become slightly curved.

Problem:

A single acting reciprocating pump has a piston of 12 cm diameter and a stroke of 30 cm. The length and diameter of the suction pipe are 10 m and 8 cm respectively. If the height of the axis of the cylinder is 3m above the water level in the sump, find the absolute pressure in the cylinder at the beginning of the suction stroke. The pump is running at 30 r.p.m. Atmospheric pressure = 101.043 kN/m^2 (1.03 kgf/cm^2).

At the beginning of the suction stroke

Total Negative pressure, $H_n = H_s + H_{as} = H_s + \frac{l_s \times A}{a_s \times g}\,\omega^2 r$

$$= 3 + \frac{10 \times \pi/4 \times (0{\cdot}12)^2}{\pi/4 \times (0{\cdot}08)^2 \times 9{\cdot}81}\left[\left(\frac{30}{60} \times 2\pi\right)^2 \times 0{\cdot}15\right]$$

$$= 3 + 3{\cdot}40 = 6{\cdot}40 \text{ m}$$

Absolute pressure = Atmospheric pressure - Negative head

= 10.3 – 6.40 **= 3.90 m of water**.

Problem:

A single acting reciprocating pump has a piston diameter of 15cm and a stroke of 60 cm. The delivery pipe is 8 cm in diameter and 30 m long. If the delivery pipe is vertical, find the maximum speed at which the pump can run so that separation does not occur in the delivery stroke. Atmospheric pressure =101.043 kN/m^2 (1.03 kgf/cm^2). It may be assumed that the separation occurs at an absolute pressure of 24.525 kN/m^2 (1.03 kgf/cm^2) .

At the end of the delivery stroke

Gauge pressure

$$H_g = H_d + H_{ad} = H_d - \frac{l_d A}{a_d g} \omega^2 r$$

$$= 30 - \frac{30 \times 15^2}{8^2 \times 9{\cdot}81} \times 0{\cdot}30 \times \omega^2 = 30 - 3{\cdot}23\ \omega^2$$

Absolute pressure, $H_d' = H_{atm} + H_g = H_{atm} + H_d + H_{ad} = H_{atm} + H_d - \frac{l_d A}{a_d g} \omega^2 r$

$$= 10.3 + 30 - 3.23\ \omega^2 = 40.3 - 3.23\ \omega^2$$

Absolute vapor pressure = 24.525 kN/m^2

So, corresponding vapor pressure head = p_v/γ = 24.525/9.81 = 2.5m

For no separation.

$$40{\cdot}3 - 3{\cdot}23\ \omega^2 = 2{\cdot}50$$

$$\omega = 3{\cdot}42 \text{ rad/sec}$$

Now

$$\omega = 2\pi N/60$$

$$N = \frac{60 \times 3{\cdot}42}{2 \times \pi} = 32{\cdot}7 \text{ r.p.m.}$$

Problem:

A single acting pump has a stroke of 30cm and the diameter of the piston is 15 cm. If the axis of the of the cylinder is 5m above the level of water in the sump and 30 m below the reservoir level and the pump is working at 30 r.p.m., **find the pressure head at the beginning, middle and end of each stroke and the power required to drive the pump**.

Solution:

Given that, $L = 30$ cm, $d_{pis} = 15$ cm, $d = 7.5$ cm, $H_s = 5$m, $H_d = 30$m, $N = 30$ rpm,

$$H_a = \frac{lA}{ag}(\omega^2 r \cos\theta)$$

$$H_{as} = \frac{l_s \times A}{a_s \times g}[\omega^2 r]$$

for $\theta = 0°$ and $180°$

$$= \frac{5 \times \pi/4 \times 0{\cdot}15^2}{\pi/4 \times (0{\cdot}075)^2 \times 9{\cdot}81}\left[\left(\frac{2\pi \times 30}{60}\right)^2 \times 0{\cdot}15\right] = 3{\cdot}02 \text{ m}$$

(1) Suction stroke

At ***the beginning of the stroke***, when $\theta = 0°$,

Pressure head $= H_s + H_{as}$

$= 5 + 3.02$ **= 8.02 m (vacuum)**

At ***the middle of the stroke***, when $\theta = 90°$,

Pressure head $= H_s + H_{as}$

$= 5 + 0 =$ **5.0 m (vacuum)**

At ***the end of the stroke***, when $\theta = 180°$,

Pressure head $= H_s + H_{as}$

$= 5 - 3.02 =$ **1.98 m (vacuum)**

(2) Delivery stroke

$$H_{ad} = \frac{l_d \times A}{a_d \times g}\ [\omega^2 r]$$ **for $\theta = 0°$ and $180°$**

$$= \frac{30 \times 15^2}{7{\cdot}5^2 \times 9{\cdot}81} \times (\pi^2) \times 0{\cdot}15 = 18{\cdot}1 \text{ m}$$

At ***the beginning of the stroke***, when $\theta = 0°$,

Pressure head $= H_d + H_{ad}$

$= 30 + 18.1$ **= 48.1 m (gauge)**

At ***the middle of the stroke***, when $\theta = 90°$,

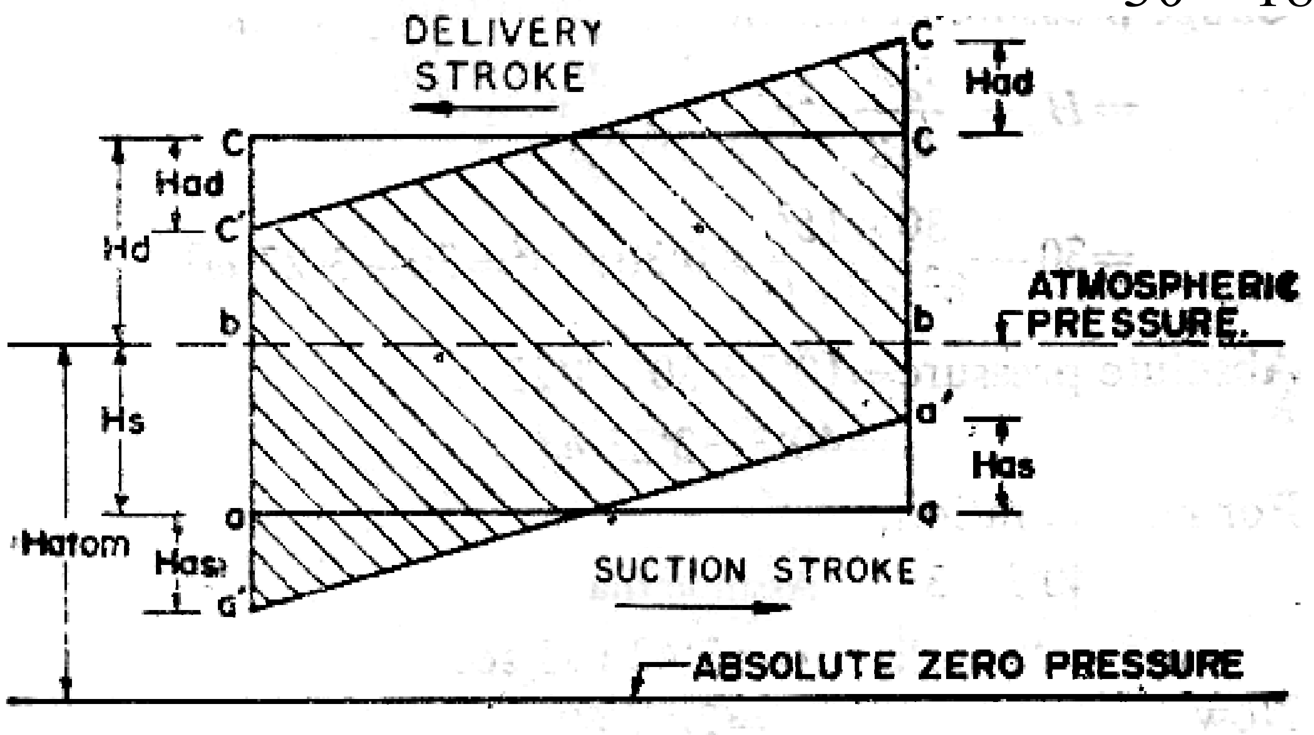
Pressure head $= H_d + H_{ad}$

$= 30 + 0$ **= 30.0 m (gauge)**

At ***the end of the stroke***, when $\theta = 180°$,

Pressure head $= H_d + H_{ad}$

$= 30 - 18.1 =$ **11.9 m (gauge)**



EFFECT OF ACCELERATION HEAD

Total area in the indicator diagram $= (H_s + H_d)\,L$

$= (5 + 30) \times 0.30$

$= 10.5$

Work done per second = γAN/60 × area in indicator diagram

$= 9.81 \times \pi/4 \times (0.15)^2 \times 30/60 \times 10.5 = 0.91$ kN-m/sec

Power required = 0.91 kW (1.22 hp)