BANGLADESH ARMY INTERNATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY (BAIUST), CUMILLA

Term Final Examination, Spring 2024

Department of Computer Science and Engineering (CSE)

Level-1, Term-1

Course Code: EEE 111 **Full Marks:** 150
Course Title: Electrical Circuit Analysis
Credit Hour: 03 **Time:** 2 hr

ANSWER SHEET

A1. i. Discuss the "Norton's theorem" how can be used to solve a linear circuit with necessary diagrams and examples.

Norton's Theorem Discussion

Statement **Norton's Theorem** states that any two-terminal linear electrical circuit can be replaced by an equivalent circuit consisting of a single **current source** $\mathbf{I_N}$ in parallel with a single **resistor** $\mathbf{R_N}$.

- The Norton Current, I_N (or short-circuit current, I_{sc}), is the current flowing through a short circuit placed across the two terminals.
- The Norton Resistance, R_N, is the equivalent resistance looking back into the two terminals when all independent voltage sources are short-circuited and all independent current sources are open-circuited. (Note: R_N is the same as the Thevenin Resistance, R_{Th}).

Procedure for Using Norton's Theorem To find the Norton equivalent circuit at terminals $a-b$:

1. **Find the Norton Current ($\mathbf{I_N}$):** Short the terminals $a-b$ and calculate the current I_{sc} flowing through the short circuit. This current is I_N.
2. **Find the Norton Resistance ($\mathbf{R_N}$):** Turn off all independent sources (replace voltage sources with short circuits and current sources with open circuits). Calculate the equivalent resistance R_{eq} looking into the terminals $a-b$. This resistance is R_N.

3. **Construct the Equivalent Circuit:** Draw the Norton equivalent circuit with the current source I_N in parallel with the resistor R_N.

4. **Connect the Load:** Reconnect the original load (if any) across the terminals $a-b$ and analyze the resulting circuit.

Diagrams (Placeholders) Figure showing the Norton Equivalent Circuit: I_N in parallel with R_N.

A1. ii. Derive the voltage equation of a capacitor and the equation of energy stored in the capacitor.

Voltage Equation of a Capacitor

The fundamental relationship between current $i(t)$ and voltage $v(t)$ for a capacitor with capacitance C is:

$$i(t) = C\frac{dv(t)}{dt}$$

To find the voltage $v(t)$, we integrate the current $i(t)$ over time. First, rearrange the equation:

$$\frac{dv(t)}{dt} = \frac{1}{C}i(t)$$

Integrate both sides from an initial time t_0 to an arbitrary time t:

$$\int_{t_0}^{t} \frac{dv(\tau)}{d\tau}d\tau = \int_{t_0}^{t} \frac{1}{C}i(\tau)d\tau$$

Applying the Fundamental Theorem of Calculus to the left side:

$$v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

Solving for the voltage $v(t)$:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

Where $v(t_0)$ is the initial voltage across the capacitor at time t_0.

Energy Stored in a Capacitor

The instantaneous power $P(t)$ absorbed by the capacitor is the product of voltage and current:

$$P(t) = v(t)i(t)$$

The energy $W(t)$ stored in the capacitor is the integral of the power over time. Assuming the capacitor was uncharged at $t = -\infty$ (i.e., $v(-\infty) = 0$):

$$W(t) = \int_{-\infty}^{t} P(\tau)d\tau = \int_{-\infty}^{t} v(\tau)i(\tau)d\tau$$

Substitute the current equation $i(t) = C\frac{dv(t)}{dt}$:

$$W(t) = \int_{-\infty}^{t} v(\tau)\left(C\frac{dv(\tau)}{d\tau}\right)d\tau$$

Change the variable of integration from time τ to voltage v:

$$W(t) = C\int_{v(-\infty)}^{v(t)} v\,dv$$

Since $v(-\infty) = 0$:

$$W(t) = C\int_{0}^{v(t)} v\,dv$$

Evaluate the integral:

$$W(t) = C\left[\frac{v^2}{2}\right]_0^{v(t)} = C\left(\frac{v^2(t)}{2} - 0\right)$$

The equation for the energy stored in the capacitor is:

$$W(t) = \frac{1}{2}Cv^2(t)$$

A2. i. Apply the superposition theorem to find $\mathbf{v_0}$ in the circuit of Figure A2.(ia) and Figure A2.(ib).

Circuit A2.(ia)

The circuit has two independent sources: a 6 A current source and an 18 V voltage source. v_0 is the voltage across the 1Ω resistor. The two 2Ω resistors are in parallel, and their equivalent resistance $R_p = 2\Omega||2\Omega = 1\Omega$. This R_p is in series with the 1Ω resistor where v_0 is measured, forming a total series resistance $R_{series} = 1\Omega + 1\Omega = 2\Omega$.

Case 1: 6 A source active, 18 V source shorted ($\mathbf{v_0'}$)

1. The 18 V source is replaced by a short circuit. The 4Ω resistor is now in parallel with $R_{series} = 2\Omega$.
2. The total 6 A current splits between $R_{series}(2\Omega)$ and $R_{4\Omega}(4\Omega)$.
3. Current through the R_{series} branch (I'), using the Current Divider Rule:

$$I' = 6\text{ A} \times \frac{R_{4\Omega}}{R_{series} + R_{4\Omega}} = 6\text{ A} \times \frac{4\Omega}{2\Omega + 4\Omega} = 6\text{ A} \times \frac{4}{6} = 4\text{ A}$$

4. This current flows through the 1Ω resistor, establishing v_0':

$$v_0' = I' \times 1\Omega = 4\text{ A} \times 1\Omega = 4\text{ V}$$

Case 2: 18 V source active, 6 A source open ($\mathbf{v_0''}$)

1. The 6 A source is replaced by an open circuit.
2. The total resistance in the loop is the series combination of 4Ω and R_{series}:

$$R_{total} = 4\Omega + 2\Omega = 6\Omega$$

3. The current flowing from the 18 V source (I''):

$$I'' = \frac{18\text{ V}}{R_{total}} = \frac{18\text{ V}}{6\Omega} = 3\text{ A}$$

4. This current flows through the 1Ω resistor, establishing v_0'':

$$v_0'' = I'' \times 1\Omega = 3\text{ A} \times 1\Omega = 3\text{ V}$$

Total Voltage $\mathbf{v_0}$

$$v_0 = v_0' + v_0'' = 4\text{ V} + 3\text{ V} = \mathbf{7}\text{ V}$$

Circuit A2.(ib)

The circuit has three independent sources connected to the common node v_0: a 5 A current source, a 9 V voltage source, and a 3 V voltage source.

Case 1: 5 A source active, 9 V shorted, 3 V shorted ($\mathbf{v_0'}$)

1. Deactivate voltage sources (short circuits). The three parallel branches have resistances 4Ω, 3Ω, and 1Ω.
2. Apply KCL at node v_0':

$$I_{\text{in}} = I_{4\Omega} + I_{3\Omega} + I_{1\Omega}$$
$$5 = \frac{v_0'}{4} + \frac{v_0'}{3} + \frac{v_0'}{1}$$
$$5 = v_0'\left(\frac{1}{4} + \frac{1}{3} + 1\right) = v_0'\left(\frac{3+4+12}{12}\right) = v_0'\frac{19}{12}$$
$$v_0' = \frac{5 \times 12}{19} = \frac{60}{19}\text{ V}$$

Case 2: 9 V source active, 5 A open, 3 V shorted ($\mathbf{v_0''}$)

1. Deactivate 5 A (open circuit) and 3 V (short circuit).
2. The circuit is now a parallel connection of 4Ω, a branch with 3Ω and 9 V (pos up), and 1Ω.
3. Apply KCL at node v_0'':

$$\frac{v_0''}{4} + \frac{v_0'' - 9}{3} + \frac{v_0''}{1} = 0$$

   Multiply by 12:

$$3v_0'' + 4(v_0'' - 9) + 12v_0'' = 0$$
$$3v_0'' + 4v_0'' - 36 + 12v_0'' = 0 \implies 19v_0'' = 36$$
$$v_0'' = \frac{36}{19}\text{ V}$$

Case 3: 3 V source active, 5 A open, 9 V shorted ($\mathbf{v_0'''}$)

1. Deactivate 5 A (open circuit) and 9 V (short circuit).
2. The circuit is a parallel connection of 4Ω, 3Ω, and a branch with 1Ω and 3 V (pos up).

3. Apply KCL at node v_0''':

$$\frac{v_0'''}{4} + \frac{v_0'''}{3} + \frac{v_0''' - 3}{1} = 0$$

Multiply by 12:

$$3v_0''' + 4v_0''' + 12(v_0''' - 3) = 0$$

$$3v_0''' + 4v_0''' + 12v_0''' - 36 = 0 \implies 19v_0''' = 36$$

$$v_0''' = \frac{36}{19}\text{ V}$$

Total Voltage $\mathbf{v_0}$

$$v_0 = v_0' + v_0'' + v_0''' = \frac{60}{19} + \frac{36}{19} + \frac{36}{19} = \frac{60 + 36 + 36}{19} = \frac{132}{19}\text{ V}$$

$$v_0 = \mathbf{\frac{132}{19}}\text{ V} \approx \mathbf{6.947}\text{ V}$$

A2. ii. Obtain the Norton equivalent of the circuit in Figure A2.(ii) to the left of terminals a − b. Use the result to find current i. (10 Marks)

The circuit to the left of terminals $a - b$ is a source network. The load is the 5Ω resistor, and i is the current flowing through it.

Step 1: Find Norton Resistance ($\mathbf{R_N}$)

Deactivate all independent sources:

- 2 A current source is replaced by an **open circuit**.
- 12 V voltage source is replaced by a **short circuit**.

Looking into terminals $a - b$:

- The 4Ω resistor is connected between the common node V_L and terminal b.
- The 6Ω resistor is connected between terminal a and node V_L (since the 12 V source is shorted).
- The open circuit from the 2 A source is in parallel with the 4Ω resistor.

Thus, $R_{6\Omega}$ and $R_{4\Omega}$ are in series between a and b:

$$R_N = 6\Omega + 4\Omega = 10\Omega$$

Step 2: Find Norton Current (I_N)

Short the terminals $a-b$ and calculate the short-circuit current I_{sc} flowing from a to b. Let $V_b=0$ and $V_a=0$. Apply Nodal Analysis at the common node V_L (left of 6Ω and 4Ω). KCL at V_L:

$$I_{2A} = I_{4\Omega} + I_{6\Omega}$$

$$2 = \frac{V_L - 0}{4} + \frac{V_L - 12 - 0}{6}$$

Multiply by 12:

$$24 = 3V_L + 2(V_L - 12)$$

$$24 = 5V_L - 24 \implies 5V_L = 48 \implies V_L = \frac{48}{5} = 9.6\text{ V}$$

The short circuit current I_{sc} is the current flowing from a to b through the 6Ω branch (where i is defined down).

$$I_N = I_{sc} = I_{6\Omega} = \frac{V_a - (V_L - 12\text{ V})}{6\Omega} \quad \text{(Current from } a \text{ to } V_L)$$

$$I_N = \frac{0 - (9.6 - 12)}{6} = \frac{2.4}{6} = 0.4\text{ A}$$

The positive sign means I_N flows from a to b.

$$I_N = \mathbf{0.4}\text{ A}$$

Step 3: Find Current i in 5Ω Load

The load resistance is $R_L = 5\Omega$. The current i is the current through R_L. Use the Current Divider Rule in the Norton equivalent circuit:

$$i = I_N \times \frac{R_N}{R_N + R_L}$$

$$i = 0.4\text{ A} \times \frac{10\Omega}{10\Omega + 5\Omega} = 0.4 \times \frac{10}{15} = 0.4 \times \frac{2}{3}$$

$$i = \frac{0.8}{3} = \frac{\mathbf{4}}{\mathbf{15}}\text{ A} \approx \mathbf{0.2667}\text{ A}$$

The current i flows **0.2667** A downwards from a to b.

A3. i. Use source transformations to reduce the circuit in Figure A3.(i) to a single voltage source in series with a single resistor.

The circuit in Figure A3.(i) consists of three parallel branches. We will convert each branch to a Norton equivalent (current source in parallel with a resistor) to find the total Norton equivalent, and then transform it to the Thevenin equivalent (voltage source in series with a resistor). Let the top node be V_A and the bottom node be V_B (reference).

Step 1: Determine Norton Equivalent Resistance (R_N)

Deactivate all independent sources: the 3 A current source is open-circuited, and the 12 V and 16 V voltage sources are short-circuited. The three resistors (10Ω, 20Ω, 40Ω) are now in parallel.

$$R_N = 10\Omega || 20\Omega || 40\Omega$$

$$\frac{1}{R_N} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = \frac{4+2+1}{40} = \frac{7}{40}$$

$$R_N = \frac{40}{7}\Omega \approx 5.714\Omega$$

The equivalent series resistance (R_{Th}) is equal to R_N.

$$R_{Th} = \frac{40}{7}\Omega$$

Step 2: Determine Norton Current (I_N)

The total Norton current I_N is the algebraic sum of the individual equivalent Norton currents (I_{N1}, I_{N2}, I_{N3}). We assume the current direction is positive when flowing UP (into node V_A).

1. **Branch 1 (3 A source || 10Ω):** The current source is flowing up.

$$I_{N1} = 3 \text{ A}$$

2. **Branch 2 (12 V source in series with 20Ω):** We assume the 12 V source polarity is consistent with the 3 A source (positive terminal towards V_A based on Nodal consistency).

$$I_{N2} = \frac{12 \text{ V}}{20\Omega} = 0.6 \text{ A} \quad \text{(Flowing up)}$$

3. **Branch 3 (16 V source in series with 40Ω):** Similarly, assume the 16 V source is oriented positive towards V_A.

$$I_{N3} = \frac{16 \text{ V}}{40\Omega} = 0.4 \text{ A} \quad \text{(Flowing up)}$$

Total Norton Current (I_N flowing up):

$$I_N = I_{N1} + I_{N2} + I_{N3} = 3 \text{ A} + 0.6 \text{ A} + 0.4 \text{ A} = 4.0 \text{ A}$$

Step 3: Determine Thevenin Voltage (V_{Th})

The Thevenin voltage is $V_{Th} = I_N \times R_N$.

$$V_{Th} = 4 \text{ A} \times \frac{40}{7}\Omega = \frac{160}{7} \text{ V}$$

Result The circuit is reduced to a single voltage source V_{Th} in series with a single resistor R_{Th}:

$$V_{Th} = \mathbf{\frac{160}{7}} \text{ V} \approx \mathbf{22.86} \text{ V}$$

$$R_{Th} = \frac{40}{7}\Omega \approx 5.71\Omega$$

A3. ii. The variable resistor R in Figure A3.(ii) is adjusted until it absorbs the maximum power from the circuit. Calculate the value of R for maximum power. Determine the maximum power absorbed by R.

We need to find the Thevenin equivalent circuit across the terminals of R.

Step 1: Calculate Thevenin Resistance ($\mathbf{R_{Th}}$)

Replace the 40 V voltage source with a short circuit. The circuit becomes a parallel combination of two series branches:

- **Upper Branch:** $80\Omega + 20\Omega = 100\Omega$
- **Lower Branch:** $10\Omega + 90\Omega = 100\Omega$

R_{Th} is the parallel combination of these two branches:

$$R_{Th} = 100\Omega||100\Omega = \frac{100 \times 100}{100 + 100} = 50\Omega$$

Value of R for Maximum Power According to the Maximum Power Transfer Theorem:

$$R = R_{Th} = 50\Omega$$

Step 2: Calculate Thevenin Voltage ($\mathbf{V_{Th}}$)

V_{Th} is the open-circuit voltage across the terminals where R was connected ($V_a - V_b$). The 40 V source is applied across the left-side internal nodes. Due to the two series loops on the bridge, the currents are:

- **Upper Loop Current ($\mathbf{I_{up}}$):** $I_{up} = \frac{40 \text{ V}}{80\Omega + 20\Omega} = 0.4 \text{ A}$
- **Lower Loop Current ($\mathbf{I_{down}}$):** $I_{down} = \frac{40 \text{ V}}{10\Omega + 90\Omega} = 0.4 \text{ A}$

The voltage at node a (V_a) is the voltage drop across the 20Ω resistor, and the voltage at node b (V_b) is the voltage drop across the 90Ω resistor (assuming the source is connected between the common nodes V_C and V_D, and we measure V_a and V_b with respect to V_D).

$$V_a = I_{up} \times 20\Omega = 0.4\text{ A} \times 20\Omega = 8\text{ V}$$

$$V_b = I_{down} \times 90\Omega = 0.4\text{ A} \times 90\Omega = 36\text{ V}$$

$$V_{Th} = V_a - V_b = 8\text{ V} - 36\text{ V} = -28\text{ V}$$

The magnitude of V_{Th} is 28 V.

Step 3: Calculate Maximum Power ($\mathbf{P_{max}}$)

$$P_{\text{max}} = \frac{V_{Th}^2}{4R_{Th}}$$

$$P_{\text{max}} = \frac{(28\text{ V})^2}{4 \times 50\Omega} = \frac{784}{200} = 3.92\text{ W}$$

Maximum Power Absorbed

$$P_{\text{max}} = \mathbf{3.92}\text{ W}$$

B4. i. Show how differentiation of a sinusoidal wave, leads to multiplication of jω with the corresponding phasor of that sinusoid.

Consider a sinusoidal voltage wave in the time domain:

$$v(t) = V_m \cos(\omega t + \phi)$$

The corresponding phasor is $\mathbf{V} = V_m \angle \phi$.

Step 1: Differentiate the Sinusoid

Differentiate $v(t)$ with respect to time:

$$\frac{dv(t)}{dt} = \frac{d}{dt}[V_m \cos(\omega t + \phi)]$$

$$\frac{dv(t)}{dt} = -V_m \omega \sin(\omega t + \phi)$$

Step 2: Convert the Resulting Sinusoid into the Cosine Reference

To convert this back into a phasor, the time-domain function must be in the form of a cosine function. We use the trigonometric identity: $-\sin(\theta) = \cos(\theta + \mathbf{90}^\circ)$ (or $\cos(\theta + \frac{\pi}{2})$).

$$\frac{dv(t)}{dt} = V_m\omega \cos(\omega t + \phi + 90^\circ)$$

Step 3: Determine the Phasor of the Derivative

The phasor of the derivative is found by taking the magnitude and phase of the resulting cosine function:

$$\text{Phasor}\left[\frac{dv(t)}{dt}\right] = (V_m\omega)\angle(\phi + 90^\circ)$$

Using Euler's identity, a phase shift of 90° corresponds to multiplication by $e^{j90^\circ} = j$:

$$\text{Phasor}\left[\frac{dv(t)}{dt}\right] = (V_m\omega)\angle\phi \cdot 1\angle 90^\circ$$

$$\text{Phasor}\left[\frac{dv(t)}{dt}\right] = (V_m\angle\phi) \cdot (\omega\angle 90^\circ)$$

Since $\omega\angle 90^\circ$ in polar form is equivalent to $j\omega$ in rectangular form:

$$\text{Phasor}\left[\frac{dv(t)}{dt}\right] = (V_m\angle\phi) \cdot (j\omega)$$

Since the phasor of the original sinusoid is $\mathbf{V} = V_m\angle\phi$:

$$\text{Phasor}\left[\frac{dv(t)}{dt}\right] = \mathbf{j}\omega\mathbf{V}$$

Thus, differentiation of a sinusoid in the time domain corresponds to multiplication of its phasor by $j\omega$.

B4. ii. Transform these sinusoids to phasors

a) $\mathbf{i} = \mathbf{6}\sin(\mathbf{50t} - \mathbf{40^0})$ **A**

The phasor is derived from the cosine function. We must use the identity $\sin(\theta) = \cos(\theta - \mathbf{90}^\circ)$.

$$i(t) = 6\cos(50t - 40^\circ - 90^\circ) = 6\cos(50t - 130^\circ) \text{ A}$$

The phasor **I** has a magnitude $I_m = 6$ A and phase $\phi = -130^\circ$:

$$\mathbf{I} = \mathbf{6}\angle - \mathbf{130}^\circ \text{ A}$$

$$\textbf{b) } \mathbf{i} = -\mathbf{4}\sin(\mathbf{30t} + \mathbf{50^0})\ \mathbf{A}$$

First, eliminate the negative sign using $-\sin(\theta) = \sin(\theta + \mathbf{180}°)$:

$$i(t) = 4\sin(30t + 50° + 180°) = 4\sin(30t + 230°)\ \text{A}$$

Now, convert to cosine using $\sin(\theta) = \cos(\theta - \mathbf{90}°)$:

$$i(t) = 4\cos(30t + 230° - 90°) = 4\cos(30t + 140°)\ \text{A}$$

The phasor $\mathbf{I}$ has a magnitude $I_m = 4$ A and phase $\phi = 140°$:

$$\mathbf{I} = \mathbf{4}\angle\mathbf{140}°\ \text{A}$$

B4. iii. Determine the sum of two phasors, we calculated in B4(ii).

We need to calculate $\mathbf{I_{sum}} = \mathbf{I_a} + \mathbf{I_b}$.

Step 1: Convert Phasors to Rectangular Form

Phasor $\mathbf{I_a}$:

$$\mathbf{I_a} = 6\angle - 130°\ \text{A}$$

$$\mathbf{I_a} = 6(\cos(-130°) + j\sin(-130°))$$
$$\mathbf{I_a} \approx 6(-0.6428 - j0.7660) \approx -3.8568 - j4.5960\ \text{A}$$

Phasor $\mathbf{I_b}$:

$$\mathbf{I_b} = 4\angle 140°\ \text{A}$$

$$\mathbf{I_b} = 4(\cos(140°) + j\sin(140°))$$
$$\mathbf{I_b} \approx 4(-0.7660 + j0.6428) \approx -3.0640 + j2.5712\ \text{A}$$

Step 2: Add the Rectangular Components

$$\mathbf{I_{sum}} = \mathbf{I_a} + \mathbf{I_b} = (-3.8568 - 3.0640) + j(-4.5960 + 2.5712)$$
$$\mathbf{I_{sum}} \approx -6.9208 - j2.0248\ \text{A}$$

Step 3: Convert the Sum back to Polar Form

Magnitude I_m:

$$I_m = \sqrt{(-6.9208)^2 + (-2.0248)^2} \approx \sqrt{47.909 + 4.100} \approx \sqrt{52.009} \approx 7.2117\ \text{A}$$

Phase ϕ: Since the real part is negative and the imaginary part is negative (Quadrant III):

$$\alpha = \tan^{-1}\left(\frac{|-2.0248|}{|-6.9208|}\right) \approx \tan^{-1}(0.2925) \approx 16.30^\circ$$

$$\phi = -180^\circ + \alpha \approx -180^\circ + 16.30^\circ = -163.70^\circ$$

or

$$\phi = 180^\circ + 16.30^\circ = 196.30^\circ$$

$$\mathbf{I_{sum}} \approx \mathbf{7.212\angle -163.7^\circ} \text{ A} \quad (\text{or } 7.212\angle 196.3^\circ \text{ A})$$

B5. i. Determine $i(t)$ in the following circuit in Figure B5.(i) using nodal analysis

The circuit components must first be converted from the time domain to the phasor domain.

Step 1: Convert to Phasor/Frequency Domain

Given the source $v_s(t) = 2\cos(10t)$ V, the angular frequency is $\omega = 10$ rad/s.

- **Voltage Source:** $\mathbf{V}_s = 2\angle 0^\circ$ V
- **Resistors:** $R_1 = 1\Omega$, $R_2 = 1\Omega$ (Impedance $\mathbf{Z}_R = R$)
- **Inductor:** $L = 1$ H

$$\mathbf{Z}_L = j\omega L = j(10)(1) = j10\Omega$$

- **Capacitor:** $C = 1$ F

$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(10)(1)} = -j0.1\Omega$$

Step 2: Apply Nodal Analysis

Define the unknown node voltage as $\mathbf{V}_1$. The 1Ω resistor on the left is in series with the voltage source, but the central 1Ω resistor is connected to the reference ground. Apply KCL at node $\mathbf{V}_1$ (sum of currents leaving the node equals zero):

$$\frac{\mathbf{V}_1 - 2\angle 0^\circ}{1} + \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1}{-j0.1} = 0$$

Group terms containing $\mathbf{V}_1$:

$$\mathbf{V}_1\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{j10}+\frac{1}{-j0.1}\right)=\frac{2}{1}$$

$$\mathbf{V}_1\left(2-j\frac{1}{10}+j\frac{1}{0.1}\right)=2$$

$$\mathbf{V}_1\left(2-j0.1+j10\right)=2$$

$$\mathbf{V}_1(2+j9.9)=2$$

Step 3: Solve for $\mathbf{V}_1$

$$\mathbf{V}_1=\frac{2}{2+j9.9}$$

Convert the denominator to polar form:

$$2+j9.9=\sqrt{2^2+9.9^2}\angle\tan^{-1}\left(\frac{9.9}{2}\right)\approx 10.10\angle 78.58^\circ$$

$$\mathbf{V}_1=\frac{2\angle 0^\circ}{10.10\angle 78.58^\circ}\approx 0.198\angle -78.58^\circ \text{ V}$$

Step 4: Solve for i

The current i is the current flowing through the central 1Ω resistor, which is $\frac{\mathbf{V}_1}{1\Omega}$.

$$\mathbf{I}=\frac{\mathbf{V}_1}{1\Omega}\approx 0.198\angle -78.58^\circ \text{ A}$$

Step 5: Convert back to Time Domain

The time domain current $i(t)$ is:

$$i(t)=0.198\cos(10t-78.58^\circ)\text{ A}$$

B5. ii. Determine voltage V_0 in the following circuit in Figure B5.(ii) using mesh analysis

The circuit is already in the phasor domain. We define three mesh currents, $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ (clockwise).

Step 1: Define Mesh Equations (Phasor Domain)

Mesh 1 (Contains current source): The $4\angle 0^\circ$ A current source dictates the relationship between $\mathbf{I}_1$ and $\mathbf{I}_3$:

$$\mathbf{I}_1 - \mathbf{I}_3 = 4\angle 0^\circ \text{ A}$$

Mesh 2 (No source, KVL):

$$(8)\mathbf{I}_2 + (15)(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$23\mathbf{I}_2 - 15\mathbf{I}_3 = 0 \quad \text{(Eq. 2)}$$

Mesh 3 (Contains voltage source and current source, KVL):

$$(12)(\mathbf{I}_3 - \mathbf{I}_1) + (15)(\mathbf{I}_3 - \mathbf{I}_2) + 10\angle 0^\circ = 0$$

$$-12\mathbf{I}_1 - 15\mathbf{I}_2 + (12 + 15)\mathbf{I}_3 = -10$$

$$-12\mathbf{I}_1 - 15\mathbf{I}_2 + 27\mathbf{I}_3 = -10 \quad \text{(Eq. 3)}$$

Step 2: Solve the System of Equations

From Eq. 1: $\mathbf{I}_1 = 4 + \mathbf{I}_3$.
Substitute $\mathbf{I}_1$ into Eq. 3:

$$-12(4 + \mathbf{I}_3) - 15\mathbf{I}_2 + 27\mathbf{I}_3 = -10$$

$$-48 - 12\mathbf{I}_3 - 15\mathbf{I}_2 + 27\mathbf{I}_3 = -10$$

$$15\mathbf{I}_3 - 15\mathbf{I}_2 = 38 \quad \text{(Eq. 4)}$$

From Eq. 2: $\mathbf{I}_2 = \frac{15}{23}\mathbf{I}_3$.
Substitute $\mathbf{I}_2$ into Eq. 4:

$$15\mathbf{I}_3 - 15\left(\frac{15}{23}\mathbf{I}_3\right) = 38$$

$$15\mathbf{I}_3\left(1 - \frac{15}{23}\right) = 38$$

$$15\mathbf{I}_3\left(\frac{23 - 15}{23}\right) = 38$$

$$15\mathbf{I}_3\left(\frac{8}{23}\right) = 38 \implies \mathbf{I}_3 = \frac{38 \times 23}{15 \times 8} = \frac{874}{120} \approx 7.283 \text{ A}$$

Step 3: Determine $\mathbf{V}_0$

The voltage $\mathbf{V}_0$ is the voltage across the 8Ω resistor, which is $\mathbf{V}_0 = 8\mathbf{I}_2$. First, find $\mathbf{I}_2$:

$$\mathbf{I}_2 = \frac{15}{23}\mathbf{I}_3 = \frac{15}{23} \times \frac{874}{120} = \frac{15 \times 874}{23 \times 120} = \frac{13110}{2760} = \frac{485}{92} \approx 5.272 \text{ A}$$

Now, find $\mathbf{V}_0$:

$$\mathbf{V}_0 = 8\mathbf{I}_2 = 8 \times \frac{485}{92} = \frac{3880}{92} = \frac{970}{23}$$

$$\mathbf{V}_0 \approx \mathbf{42.174\angle 0^\circ} \text{ V}$$

(Since all sources and impedances are real, the voltage $\mathbf{V}_0$ is real and has a phase of 0°).

B6. i. Use superposition theorem to find $\mathbf{i}_o$ in the following circuit in Figure B6.(i).

The circuit has two independent sources: a current source $\mathbf{I}_s = 1\angle 0^\circ$ A and a voltage source $\mathbf{V}_s = 20\angle 90^\circ$ V. We find the load current $\mathbf{i}_o$ (through the $j10\Omega$ inductor) by summing the results from two separate cases.

Step 1: Define Impedances (Phasor Domain)

The circuit components are given as steady-state impedances:

- Resistors: $R_1 = 4\Omega$, $R_2 = 20\Omega$, $R_3 = 1\Omega$
- Inductor: $\mathbf{Z}_L = j10\Omega$
- Capacitor: $\mathbf{Z}_C = -j5\Omega$

Case 1: $1\angle 0^\circ$ A Current Source Active, $\mathbf{V}_s$ Shorted ($\mathbf{i}'_o$)

The voltage source is short-circuited. The 4Ω resistor is now in parallel with the 1Ω resistor.

1. **Equivalent Impedance (R_{eq}):** The 4Ω and 1Ω resistors are in parallel due to the short circuit.

$$R_{eq} = 4\Omega || 1\Omega = \frac{4 \times 1}{4 + 1} = 0.8\Omega$$

2. **Impedance of the Load Branch ($\mathbf{Z}_{loop}$):** The current flows through R_{eq}, $\mathbf{Z}_L$, and $\mathbf{Z}_C$ in series.

$$\mathbf{Z}_{loop} = R_{eq} + \mathbf{Z}_L + \mathbf{Z}_C = 0.8 + j10 - j5 = 0.8 + j5\Omega$$

3. **Current Divider Rule (CDR):** The $1\angle 0^\circ$ A current splits between the 20Ω resistor and $\mathbf{Z}_{loop}$. The current flowing into the loop is $\mathbf{i}'_o$:

$$\mathbf{i}'_o = 1\angle 0^\circ \cdot \frac{R_2}{R_2 + \mathbf{Z}_{loop}} = \frac{20}{20 + 0.8 + j5} = \frac{20}{20.8 + j5}$$

4. **Calculation:** Convert the denominator to polar form: $20.8 + j5 \approx 21.39\angle 13.5^\circ$.

$$\mathbf{i}'_o \approx \frac{20\angle 0^\circ}{21.39\angle 13.5^\circ} \approx \mathbf{0.935\angle - 13.5^\circ} \text{ A}$$

Case 2: $20\angle 90^\circ$ V Voltage Source Active, $\mathbf{I}_s$ Opened ($\mathbf{i}''_o$)

The current source is replaced by an open circuit, disconnecting the 20Ω resistor.

1. **Total Loop Impedance ($\mathbf{Z}_{total}$):** The components R_1, R_3, $\mathbf{Z}_L$, and $\mathbf{Z}_C$ are now in series with $\mathbf{V}_s$.

$$\mathbf{Z}_{total} = R_1 + R_3 + \mathbf{Z}_L + \mathbf{Z}_C = 4 + 1 + j10 - j5 = 5 + j5\Omega$$

2. **Loop Current ($\mathbf{i}''_o$):** The current is found using Ohm's Law.

$$\mathbf{i}''_o = \frac{\mathbf{V}_s}{\mathbf{Z}_{total}} = \frac{20\angle 90^\circ}{5 + j5}$$

3. **Calculation:** Convert the denominator to polar form: $5 + j5 \approx 7.071\angle 45^\circ$.

$$\mathbf{i}''_o = \frac{20\angle 90^\circ}{7.071\angle 45^\circ} \approx \mathbf{2.828\angle 45^\circ} \text{ A}$$

Step 3: Total Current $\mathbf{i}_o$

The total current is the phasor sum $\mathbf{i}_o = \mathbf{i}'_o + \mathbf{i}''_o$.

1. **Convert to Rectangular Form:**

$$\mathbf{i}'_o \approx 0.935\cos(-13.5^\circ) + j0.935\sin(-13.5^\circ) \approx 0.908 - j0.218 \text{ A}$$

$$\mathbf{i}''_o \approx 2.828\cos(45^\circ) + j2.828\sin(45^\circ) \approx 2.000 + j2.000 \text{ A}$$

2. **Summation:**

$$\mathbf{i}_o \approx (0.908 + 2.000) + j(-0.218 + 2.000) \approx 2.908 + j1.782 \text{ A}$$

3. **Final Polar Form:**

$$I_m = \sqrt{2.908^2 + 1.782^2} \approx 3.41 \text{ A}$$

$$\phi = \tan^{-1}\left(\frac{1.782}{2.908}\right) \approx 31.46^\circ$$

Final Answer

$$\mathbf{i}_o \approx \mathbf{3.41\angle 31.46^\circ}\ \text{A}$$

B6. ii. At $\omega = 10^3 rad/s$ find the input admittance of the following circuit in Figure B6.(ii).

The input admittance, $\mathbf{Y}_{\text{in}}$, is the reciprocal of the total input impedance, $\mathbf{Z}_{\text{in}}$, where $\mathbf{Z}_{\text{in}}$ is the series combination of the capacitor $\mathbf{Z}_C$ and the equivalent parallel impedance $\mathbf{Z}_{\text{eq}}$.

Step 1: Calculate Impedances at $\omega = \mathbf{10^3}$ rad/s

[cite$_{start}$]$The circuit contains a capacitor ($C=20 μF [cite: 124][cite$_{start}$]$) and an inductor ($L=10 mH$[cite: 130]])$.

1. **Capacitor Impedance ($\mathbf{Z}_C$):** (Series component)

$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(10^3)(20 \times 10^{-6})} = \frac{1}{j(0.02)} = -j50\Omega$$

2. **Inductor Impedance ($\mathbf{Z}_L$):**

$$\mathbf{Z}_L = j\omega L = j(10^3)(10 \times 10^{-3}) = j10\Omega$$

Step 2: Calculate Equivalent Admittance ($\mathbf{Y}_{\text{eq}}$)

The parallel section, $\mathbf{Z}_{\text{eq}}$, consists of three main branches connected from the central top node to the bottom reference node. We assume the structure is a parallel combination of 30Ω, 60Ω, and the series combination of 40Ω and $\mathbf{Z}_L$.

$$\mathbf{Y}_{\text{eq}} = \frac{1}{30} + \frac{1}{60} + \frac{1}{40 + \mathbf{Z}_L}$$

1. **Admittance of Resistors ($\mathbf{Y}_R$):**

$$\mathbf{Y}_R = \frac{1}{30} + \frac{1}{60} = \frac{2+1}{60} = \frac{3}{60} = 0.05 \text{ S}$$

2. **Admittance of $\mathbf{Z}_{40+L}$ ($\mathbf{Y}_L$):**

$$\mathbf{Y}_L = \frac{1}{40 + j10} = \frac{1}{40 + j10} \times \frac{40 - j10}{40 - j10} = \frac{40 - j10}{1600 + 100} = \frac{40 - j10}{1700}$$

$$\mathbf{Y}_L \approx 0.02353 - j0.00588 \text{ S}$$

3. **Total Parallel Admittance ($\mathbf{Y}_{\text{eq}}$):**

$$\mathbf{Y}_{\text{eq}} = \mathbf{Y}_R + \mathbf{Y}_L \approx 0.05 + 0.02353 - j0.00588$$

$$\mathbf{Y}_{\text{eq}} \approx 0.07353 - j0.00588 \text{ S}$$

Step 3: Calculate Total Input Impedance ($\mathbf{Z}_{\text{in}}$)

$$\mathbf{Z}_{\text{eq}} = \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{0.07353 - j0.00588}$$
$$\mathbf{Z}_{\text{eq}} \approx 13.516 + j1.081\Omega$$

The total input impedance $\mathbf{Z}_{\text{in}}$ is the series combination of $\mathbf{Z}_C$ and $\mathbf{Z}_{\text{eq}}$:

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_C + \mathbf{Z}_{\text{eq}} = -j50 + (13.516 + j1.081)$$

$$\mathbf{Z}_{\text{in}} \approx 13.516 - j48.919\Omega$$

Step 4: Calculate Input Admittance ($\mathbf{Y}_{\text{in}}$)

$$\mathbf{Y}_{\text{in}} = \frac{1}{\mathbf{Z}_{\text{in}}} = \frac{1}{13.516 - j48.919}$$

Rationalize the denominator:

$$\mathbf{Y}_{\text{in}} = \frac{1}{13.516 - j48.919} \times \frac{13.516 + j48.919}{13.516 + j48.919}$$

$$\mathbf{Y}_{\text{in}} = \frac{13.516 + j48.919}{(13.516)^2 + (48.919)^2} \approx \frac{13.516 + j48.919}{2575.7}$$
$$\mathbf{Y}_{\text{in}} \approx 0.00525 + j0.0190 \text{ S}$$

Final Answer in Polar Form (Optional)

$$\mathbf{Y}_{\text{in}} \approx \sqrt{0.00525^2 + 0.0190^2} \angle \tan^{-1}\left(\frac{0.0190}{0.00525}\right)$$

$$\mathbf{Y}_{\text{in}} \approx \mathbf{0.0197\angle 74.55^\circ} \text{ S}$$

B7. i. Find v_o of the circuit shown in Figure B7.(i) using superposition theorem.

The circuit has three independent sources operating at three different frequencies:

- $\mathbf{V}_{s1} : 10\cos(2t)$ V $\implies \omega_1 = 2$ rad/s
- $\mathbf{I}_{s2} : 2\sin(5t)$ A $\implies \omega_2 = 5$ rad/s
- $\mathbf{V}_{s3} : 5$ V (DC) $\implies \omega_3 = 0$ rad/s

The total output voltage is the sum of the time-domain components: $v_o(t) = v_{o1}(t) + v_{o2}(t) + v_{o3}(t)$. We define the output voltage as $\mathbf{V}_o = \mathbf{V}_a - \mathbf{V}_b$, where $\mathbf{V}_a$ is the node voltage between the inductor (2 H) and the resistor (1Ω), and $\mathbf{V}_b$ is the node voltage between the resistor (1Ω) and the capacitor (0.1 F).

Case 1: $10\cos(2t)$ **V Source Active (**$\omega_1 = 2$ **rad/s)**

$\mathbf{V}_{s1} = 10\angle 0^\circ$ V. $\mathbf{I}_{s2}$ is opened, $\mathbf{V}_{s3}$ is shorted.

Impedances at $\omega_1 = 2$**:**

- $\mathbf{Z}_{L1} = j\omega_1 L = j(2)(2) = j4\Omega$
- $\mathbf{Z}_{C1} = \frac{1}{j\omega_1 C} = \frac{1}{j(2)(0.1)} = -j5\Omega$

Applying Nodal Analysis at $\mathbf{V}_a$ and $\mathbf{V}_b$ (with $\mathbf{V}_{s3}$ shorted, 4Ω is to ground):

$$\text{KCL at } \mathbf{V}_a: \quad \frac{\mathbf{V}_a - 10}{j4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{1} = 0 \implies \mathbf{V}_a(1 - j0.25) - \mathbf{V}_b = -j2.5$$
$$\text{KCL at } \mathbf{V}_b: \quad \frac{\mathbf{V}_b - \mathbf{V}_a}{1} + \frac{\mathbf{V}_b}{-j5} + \frac{\mathbf{V}_b}{4} = 0 \implies -\mathbf{V}_a + \mathbf{V}_b(1.25 + j0.2) = 0$$

Solving the system of equations yields $\mathbf{V}_{o1} = \mathbf{V}_a - \mathbf{V}_b$:

$$\mathbf{V}_{o1} \approx 2.013 - j1.664 \text{ V} \approx 2.61\angle -39.55^\circ \text{ V}$$

$$v_{o1}(t) \approx 2.61\cos(2t - 39.55^\circ) \text{ V}$$

Case 2: $2\sin(5t)$ **A Source Active (**$\omega_2 = 5$ **rad/s)**

$\mathbf{I}_{s2} = 2\angle -90^\circ = -j2$ A. $\mathbf{V}_{s1}$ and $\mathbf{V}_{s3}$ are shorted.

Impedances at $\omega_2 = 5$**:**

- $\mathbf{Z}_{L2} = j\omega_2 L = j(5)(2) = j10\Omega$
- $\mathbf{Z}_{C2} = \frac{1}{j\omega_2 C} = \frac{1}{j(5)(0.1)} = -j2\Omega$

Applying Nodal Analysis ($\mathbf{V}_{s1}$ shorted, $\mathbf{V}_{s3}$ shorted):

$$\text{KCL at } \mathbf{V}_a: \quad \frac{\mathbf{V}_a}{j10} + \frac{\mathbf{V}_a - \mathbf{V}_b}{1} = -(-j2) \implies \mathbf{V}_a(1 - j0.1) - \mathbf{V}_b = j2$$
$$\text{KCL at } \mathbf{V}_b: \quad \frac{\mathbf{V}_b - \mathbf{V}_a}{1} + \frac{\mathbf{V}_b}{-j2} + \frac{\mathbf{V}_b}{4} = 0 \implies -\mathbf{V}_a + \mathbf{V}_b(1.25 + j0.5) = 0$$

Solving the system of equations yields $\mathbf{V}_{o2} = \mathbf{V}_a - \mathbf{V}_b$:

$$\mathbf{V}_{o2} \approx -0.488 + j2.2765 \text{ V} \approx 2.328\angle 102.08^\circ \text{ V}$$

Converting back to the time domain (cosine reference):

$$v_{o2}(t) \approx 2.328\cos(5t + 102.08^\circ) \text{ V}$$

Case 3: 5 **V DC Source Active (**$\omega_3 = 0$**)**

$\mathbf{V}_{s1}$ is shorted, $\mathbf{I}_{s2}$ is opened.

- $\mathbf{Z}_{L3} = 0\Omega$ (short circuit)
- $\mathbf{Z}_{C3} = \infty\Omega$ (open circuit)

The shorted inductor (in series with the shorted $\mathbf{V}_{s1}$) connects $\mathbf{V}_a$ directly to the ground reference. Thus, $\mathbf{V}_a = 0$ V. The open capacitor forces the current in the right loop (containing 1Ω, 4Ω, and 5 V) to zero.

$$\text{KCL at } \mathbf{V}_b : \frac{\mathbf{V}_b - \mathbf{V}_a}{1} + i_C = 0$$

Since $\mathbf{V}_a = 0$ and $i_C = 0$, this requires $\mathbf{V}_b = 0$ V.

$$v_{o3} = \mathbf{V}_a - \mathbf{V}_b = 0 - 0 = 0 \text{ V}$$

Final Summation

$$v_o(t) = v_{o1}(t) + v_{o2}(t) + v_{o3}(t)$$

$$v_o(t) \approx \mathbf{2.61}\cos(\mathbf{2t} - \mathbf{39.55}^\circ) + \mathbf{2.328}\cos(\mathbf{5t} + \mathbf{102.08}^\circ) \text{ V}$$

B7. ii. Obtain the Thevenin impedence at terminals a-b of the following circuit in Figure B7.(ii).

To find the Thevenin impedance ($\mathbf{Z}_{\text{Th}}$), we replace the independent voltage source ($\mathbf{V}_s = 120\angle 75^\circ$ V) with a short circuit. The terminals 'a' and 'b' are within a bridge network, and the source is connected across the input terminals of the bridge ('d' and 'c').

Step 1: Simplify the Circuit

When the source $\mathbf{V}_s$ is shorted, terminals 'd' and 'c' are connected together.

- Impedance $\mathbf{Z}_1 = -j6\Omega$ (between d and a) is in parallel with $\mathbf{Z}_3 = 8\Omega$ (between c and a).
- Impedance $\mathbf{Z}_2 = 4\Omega$ (between d and b) is in parallel with $\mathbf{Z}_4 = j12\Omega$ (between c and b).

The Thevenin impedance $\mathbf{Z}_{\text{Th}}$ seen between a and b is the series combination of these two parallel groups:

$$\mathbf{Z}_{\text{Th}} = (\mathbf{Z}_1 || \mathbf{Z}_3) + (\mathbf{Z}_2 || \mathbf{Z}_4)$$

Step 2: Calculate $\mathbf{Z}_{a,c/d}$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 || \mathbf{Z}_3 = \frac{(-j6)(8)}{-j6+8} = \frac{-j48}{8-j6}$$

$$\mathbf{Z}_{ac} = \frac{-j48}{8-j6} \times \frac{8+j6}{8+j6} = \frac{-j384+288}{64+36} = \frac{288-j384}{100}$$

$$\mathbf{Z}_{ac} = 2.88 - j3.84\Omega$$

Step 3: Calculate $\mathbf{Z}_{b,c/d}$

$$\mathbf{Z}_{bc} = \mathbf{Z}_2 || \mathbf{Z}_4 = \frac{(4)(j12)}{4+j12} = \frac{j48}{4+j12}$$

$$\mathbf{Z}_{bc} = \frac{j48}{4+j12} \times \frac{4-j12}{4-j12} = \frac{j192+576}{16+144} = \frac{576+j192}{160}$$

$$\mathbf{Z}_{bc} = \frac{384}{160} + j\frac{192}{160} = 3.6 + j1.2\Omega$$

Step 4: Calculate $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \mathbf{Z}_{ac} + \mathbf{Z}_{bc} = (2.88 - j3.84) + (3.6 + j1.2)$$

$$\mathbf{Z}_{\text{Th}} = (2.88 + 3.6) + j(-3.84 + 1.2)$$

$$\mathbf{Z}_{\text{Th}} = \mathbf{6.48 - j2.64}$$