ME265: Thermal Engineering & Heat Transfer

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2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

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4.2.3 Steady Heat Conduction

For one dimensional (1D) medium with constant properties and with no heat generation:

C/System	Governing Equation	Thermal Condition	Examples
Cartesian	$\frac{d^2T}{dx^2} = 0$	Steady State	Slab, Plane wall
Cylindrical	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	Steady State	Solid & hollow pipe, tube, wire, etc.
Spherical	$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$	Steady State	Solid & hollow sphere

These governing equations will be used to illustrate **Thermal resistances**

4.2.3.2 Thermal Resistances, R

$$Q = \frac{T_1 - T_2}{R}$$

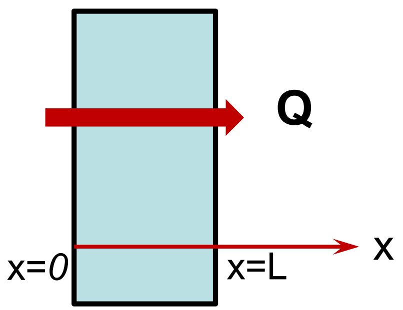
R is the thermal resistance and is associated with 1st kind BC

R depends on

- modes of heat transfer
- geometry
- properties of medium

4.2.3.2 Thermal Resistances: for conduction, R_{cond}

1D-SS heat conduction in solids with 1st kind (temperature) BC



Plane wall

Math model

$$\frac{d^2T}{dx^2} = 0 \;\ldots\ldots \textit{for } 0<x<L$$

$$T = T_1 \;\ldots\ldots \text{at } x=0$$

$$T = T_2 \;\ldots\ldots \text{at } x=L$$

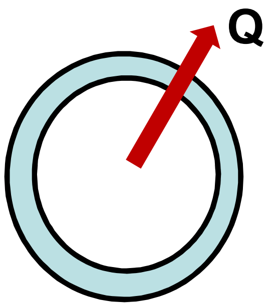
Solving the math model, we get: $\frac{dT}{dx} = C_1$ and $T = C_1 x + C_2$

$$C_2 = T_2 \text{ and } C_1 = (T_2 - T_1)/L$$

$$Q = -kA\frac{dT}{dx} = -kAC_1 = \frac{T_1 - T_2}{\frac{L}{kA}} = \frac{T_1 - T_2}{R} \Rightarrow \quad R = \frac{L}{kA}$$

4.2.3.2 Thermal Resistances for conduction, R_{cond}

1D-SS heat conduction in solids with 1st kind (temperature) BC



Hollow cylinder

Math model

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \dots \dots \text{ for } r_1<r<r_2$$

$$T = T_1 \quad \dots \dots \text{ at } r = r_1$$

$$T = T_2 \quad \dots \dots \text{ at } r = r_2$$

Solving the math model, we get:

$$\frac{dT}{dr} = \frac{C_1}{r} \;\&\; T = C_1 ln(r) + C_2$$

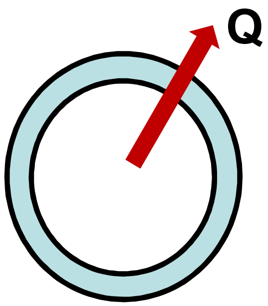
$$C_1 = -\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

$$Q = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r}$$

$$= \frac{T_1 - T_2}{\frac{ln(r_2/r_1)}{2\pi Lk}} = \frac{T_1 - T_2}{R} \Rightarrow R = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$$

4.2.3.2 Thermal Resistances: for conduction, R_{cond}

1D-SS heat conduction in solids with 1st kind (temperature) BC



Hollow sphere

Math model

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \ldots\ldots \text{ for } r_1 < r < r_2$$

$$T = T_1 \quad \ldots\ldots \text{ at } r=r_1$$

$$T = T_2 \quad \ldots\ldots \text{ at } r=r_2$$

Solving the math model, we get:

$$\frac{dT}{dr} = \frac{C_1}{r^2} \;\&\; T = -C_1/r + C_2$$

$$C_1 = -\frac{(T_1 - T_2)(r_2 - r_1)}{r_1 r_2}$$

$$Q = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2}$$

$$= \frac{T_1 - T_2}{\frac{r_2 - r_1}{4\pi r_1 r_2 k}} = \frac{T_1 - T_2}{R} \Rightarrow \quad R = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

4.2.3.2 Thermal Resistances: for conduction, R_{cond}

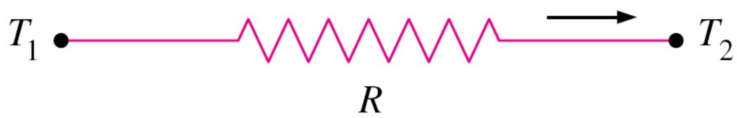
1D-SS heat conduction in solids with 1st kind (temperature) BC

Geometry	Governing Equation	Thermal Resistance, R
Wall	$\frac{d^2T}{dx^2} = 0$	$R = \frac{L}{kA}$
Hollow pipe	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$R = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$
Hollow sphere	$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$	$R = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$

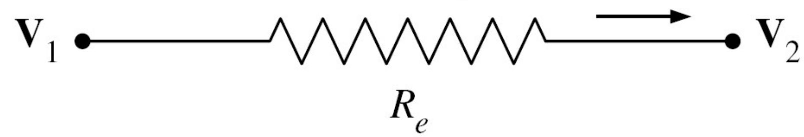
Unit of thermal resistance: $^oC/W$; $^oF.hr/Btu$

Analogy to Electrical Current Flow

$$\dot{Q} = \frac{T_1 - T_2}{R}$$



$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

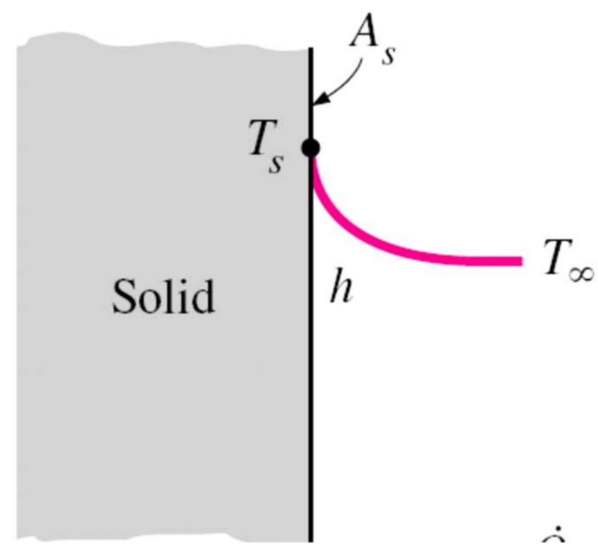


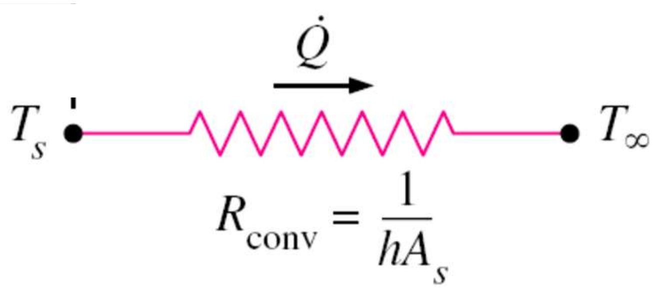
Heat Transfer		**Electrical current flow**
Rate of heat transfer	→	Electric current
Thermal resistance	→	Electrical resistance
Temperature difference	→	Voltage difference

4.2.3.2 Thermal Resistances: for convection , R_{conv}

1D-SS heat conduction in solids with 1st kind (temperature) BC

$$Q = hA(T_s - T_\infty) = \frac{(T_s - T_\infty)}{1/hA}$$





Unit of thermal resistance: $^oC/W$; $^oF.hr/Btu$

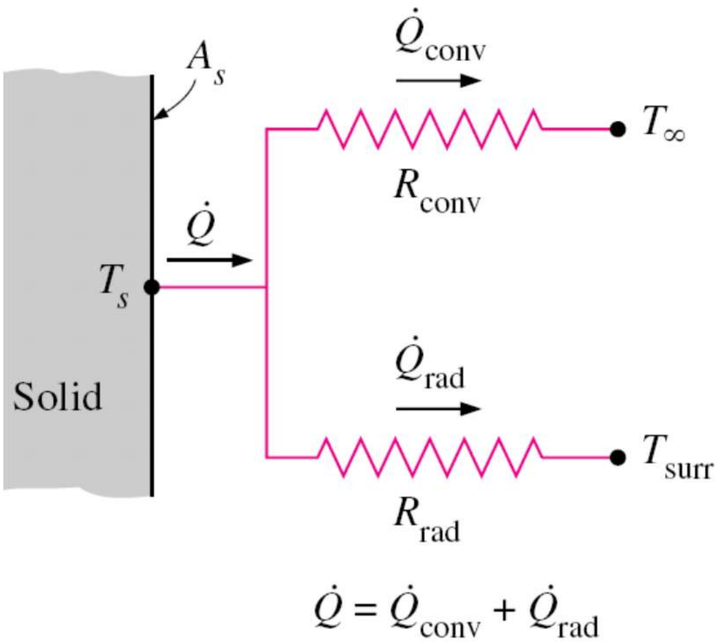
4.2.3.2 Thermal Resistances: for radiation , R_{rad}

$$\dot{Q}_{rad} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right)$$

$$= h_{rad} A_s (T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}}$$

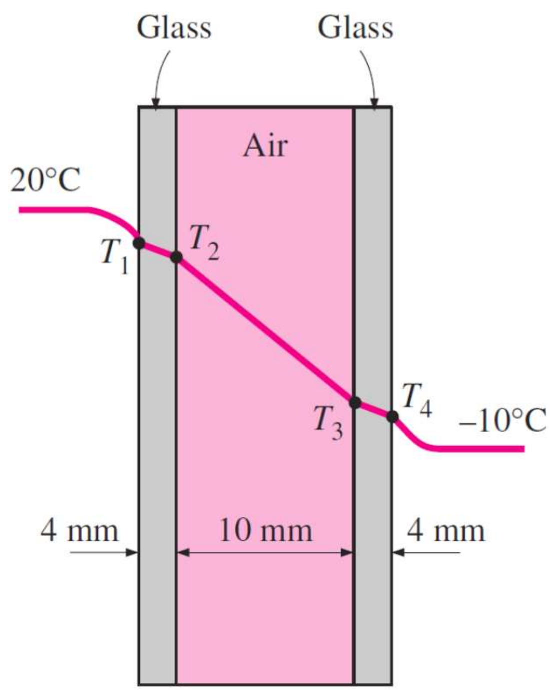
$$R_{rad} = \frac{1}{h_{rad} A_s}$$

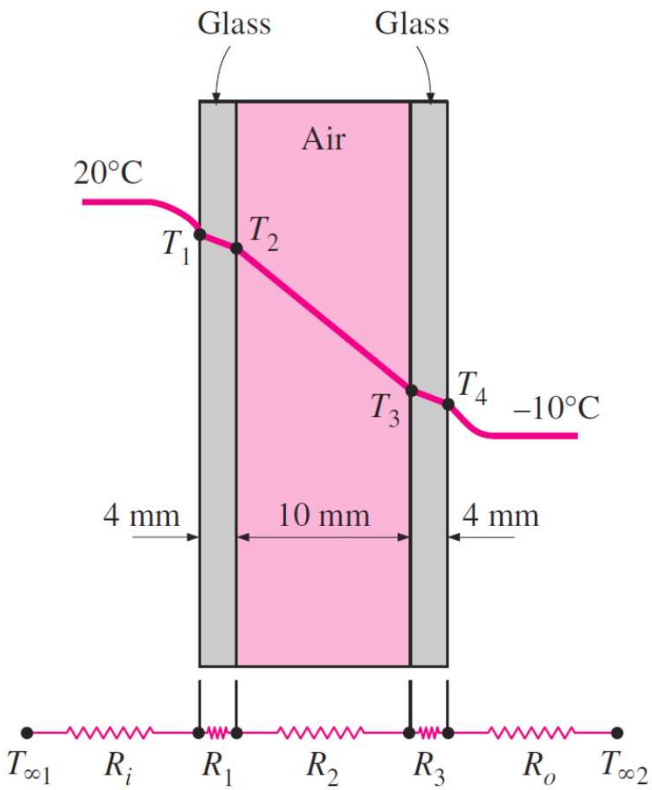
$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$



EP#2.3 Heat loss from double pane windows: (Cengel et al Example 3-3)

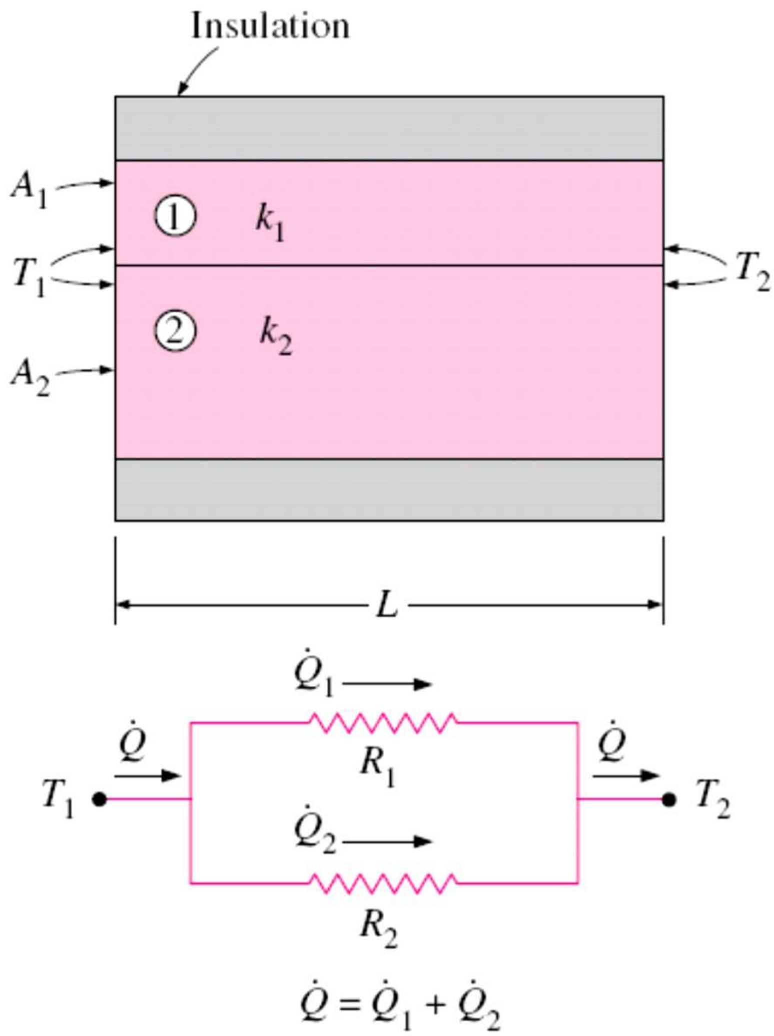
Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass (k=0.78 W/m°C) separated by a 10-mm-wide stagnant air space (k=0.026 W/m°C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be h_1=10 W/m^2°C and h_2=40 W/m^2°C, which includes the effects of radiation.



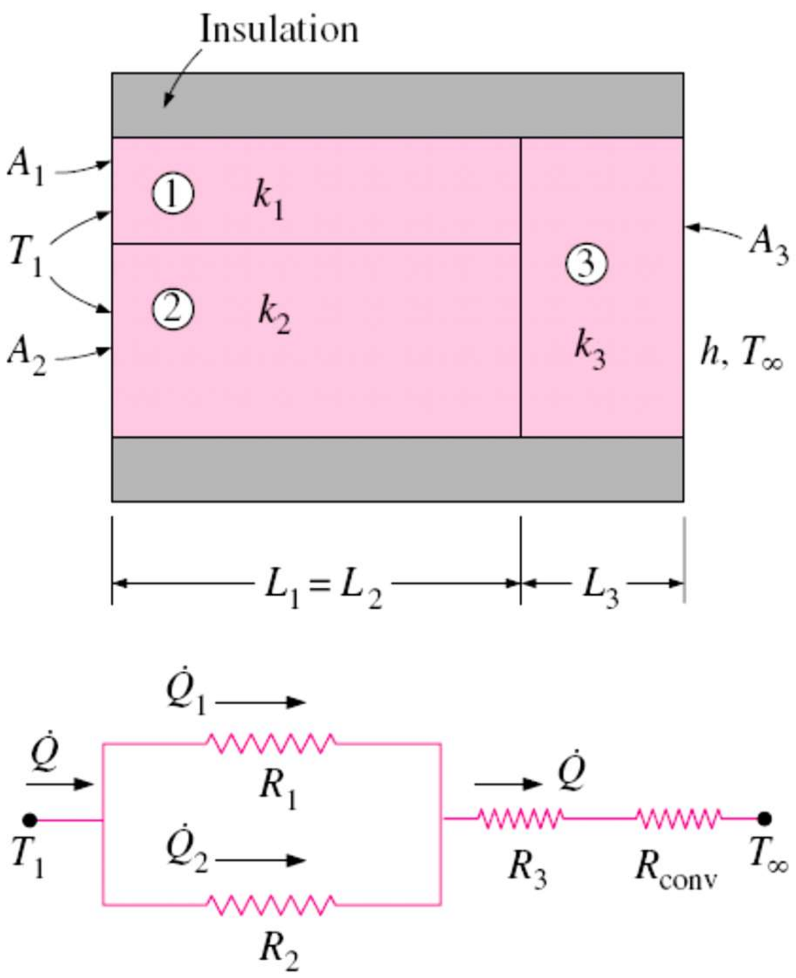


EP#2.3--Solution

4.2.3.2 Thermal Resistances: Composite solid wall

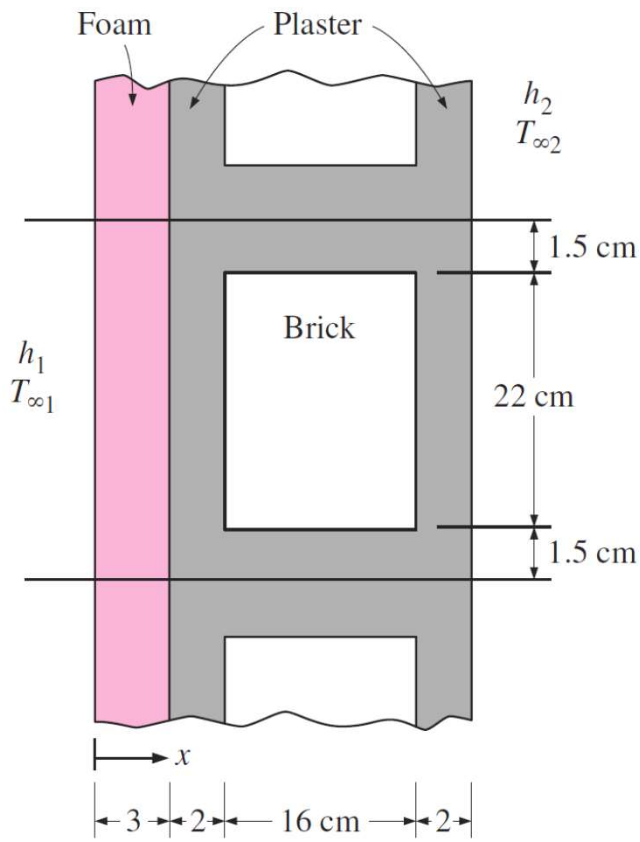


4.2.3.2 Thermal Resistances: Composite solid wall

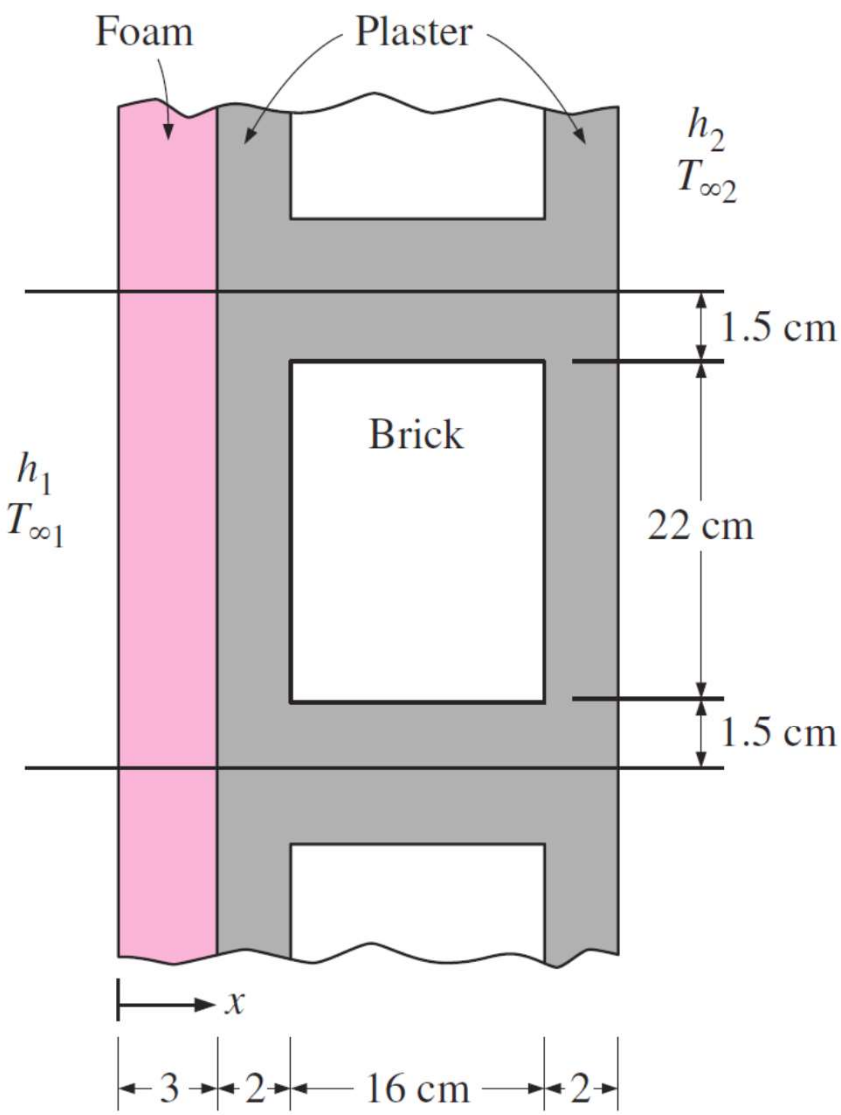


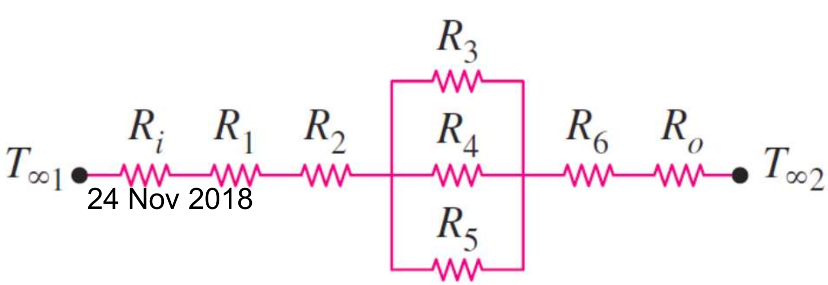
EP#2.4 Heat loss from a composite wall: (Cengel et al Example 3-6)

A 3-m-high and 5-m-wide wall consists of long 16-cm×22-cm cross section horizontal bricks (k=0.72 W/m°C) separated by 3-cm-thick plaster layers (k=0.22 W/m°C). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k=0.026 W/m°C) on the inner side of the wall. The indoor and the outdoor temperatures are 20 °C and -10 °C, and the convection heat transfer coefficients on the inner and the outer sides are h_1=10 W/m^2°C and h_2= 25 W/m^2°C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, **determine the rate of heat transfer through the wall**.



EP#2.4-Solution





EP#2.4 (Soln)