The Time Value of Money

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25-08-2020

Money has a time value.

- *Capital* refers to wealth in the form of money or property that can be used to produce more wealth.
- Engineering economy studies involve the commitment of capital for extended periods of time.
- A dollar today is worth more than a dollar one or more years from now (for several reasons).


ONE HUNDRED TRILLION DOLLARS
RESERVE

Return to capital

- Interest and profit pay the providers of capital for forgoing its use during the time the capital is being used.
- Interest and profit are payments for the *risk* the investor takes in letting another use his or her capital.
- Any project or venture must provide a sufficient return to be financially attractive to the suppliers of money or property.

Simple interest

When the total interest earned or charged is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods, the interest and interest rate are said to be *simple*.

Computation of simple interest

The total interest, I, earned or paid may be computed using the formula below.

$$\underline{I} = (P)(N)(i)$$

P = principal amount lent or borrowed

N = number of interest periods (e.g., years)

i = interest rate per interest period

The total amount repaid at the end of N interest periods is $P + \underline{I}$.

Example on simple interest

If \$5,000 were loaned for five years at a simple interest rate of 7% per year, the interest earned would be

$$\underline{I} = \$5,000 \quad x \quad 5 \quad x \quad 0.07 = \$1,750$$

So, the total amount repaid at the end of five years would be the original amount (\$5,000) plus the interest (\$1,750), or \$6,750.

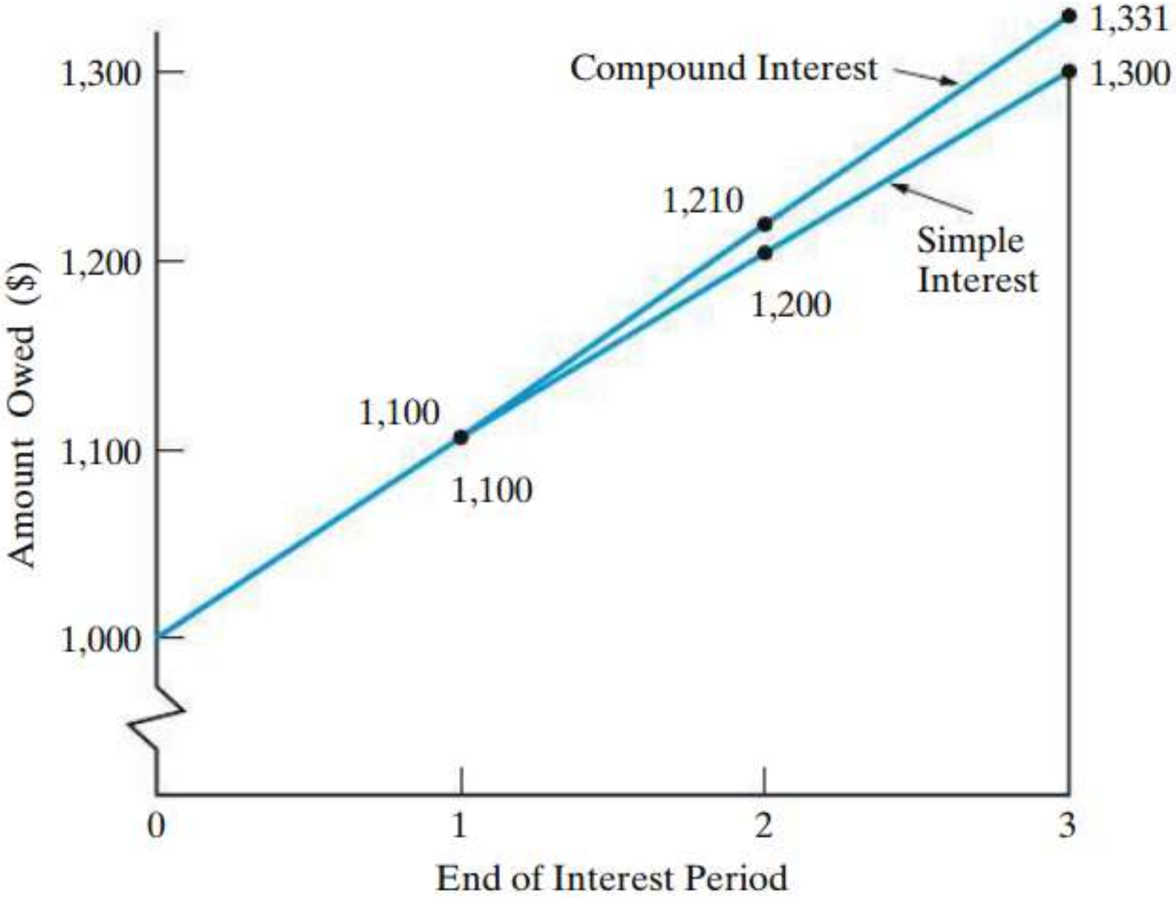
Compound interest

Compound interest reflects both the remaining principal and any accumulated interest. For $1,000 at 10%…

Period	(1) Amount owed at beginning of period	(2)=(1)x10% Interest amount for period	(3)=(1)+(2) Amount owed at end of period
1	$1,000	$100	$1,100
2	$1,100	$110	$1,210
3	$1,210	$121	$1,331

Compound interest is commonly used in personal and professional financial transactions.

Simple vs. Compound Interest



Economic Equivalence

- Economic equivalence allows us to compare alternatives on a common basis.
- Each alternative can be reduced to an *equivalent basis* dependent on
  - interest rate,
  - amount of money involved, and
  - timing of monetary receipts or expenses.
- Using these elements we can “move” cash flows so that we can compare them at particular points in time.

Economic Equivalence

suppose you have a $17,000 balance on your credit card. "This has got to stop!" you say to yourself. So you decide to repay the $17,000 debt in four months. An unpaid credit card balance at the beginning of a month will be charged interest at the rate of 1% by your credit card company. For this situation, we have selected three plans to repay the $17,000 principal plus interest owed.* These three plans are illustrated in Table 4-1, and we will demonstrate that they are equivalent (i.e., the same) when the interest rate is 1% per month on the unpaid balance of principal.

TABLE 4-1 Three Plans for Repayment of $17,000 in Four Months with Interest at 1% per Month

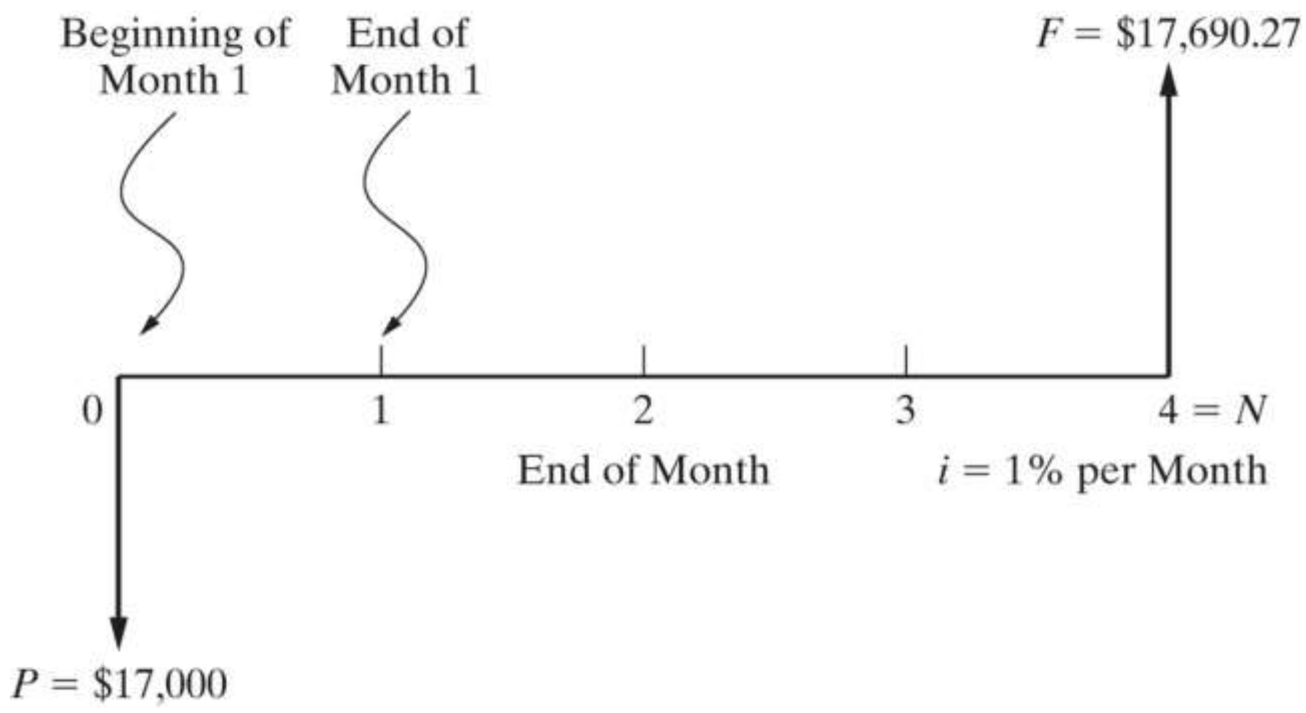
(1) Month	(2) Amount Owed at Beginning of Month	(3) = 1% × (2) Interest Accrued for Month	(4) = (2) + (3) Total Money Owed at End of Month	(5) Principal Payment	(6) = (3) + (5) Total End-of-Month Payment (Cash Flow)
Plan 1: Pay interest due at end of each month and principal at end of fourth month.					
1	$17,000	$170	$17,170	$0	$170
2	17,000	170	17,170	0	170
3	17,000	170	17,170	0	170
4	17,000	170	17,170	17,000	17,170
	68,000 $-mo.	$680 (total interest)			
Plan 2: Pay off the debt in four equal end-of-month installments (principal and interest).					
1	$17,000	$170	$17,170	$4,187.10	$4,357.10
2	12,812.90	128.13	12,941.03	4,228.97	4,357.10
3	8,583.93	85.84	8,669.77	4,271.26	4,357.10
4	4,312.67	43.13	4,355.80	4,313.97	4,357.10
	42,709.5 $-mo.	$427.10 (total interest)	Difference = $1.30 due to roundoff		
Plan 3: Pay principal and interest in one payment at end of fourth month.*					
1	$17,000	$170	$17,170	$0	$0
2	17,170	171.70	17,341.70	0	0
3	17,341.70	173.42	17,515.12	0	0
4	17,515.12	175.15	17,690.27	17,000	17,690.27
	69,026.8 $-mo.	$690.27 (total interest)			

Tools for economic equivalence.

- Notation used in formulas for compound interest calculations.
  - i = effective interest rate per interest period
  - N = number of compounding (interest) periods
  - P = present sum of money; *equivalent* value of one or more cash flows at a reference point in time; the present
  - F = future sum of money; *equivalent* value of one or more cash flows at a reference point in time; the future
  - A = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

Cash Flow Diagram

A cash flow diagram is an indispensable tool for clarifying and visualizing a series of cash flows.



Cash Flow Tables

Cash flow tables are essential to modeling engineering economy problems in a spreadsheet

= – 25000 – 9400

= C3 – B3

= SUM(D$3:D3)

	A	B	C	D	E
1		Alternative *A*	Alternative *B*	Difference	Cumulative
2	End of Year	Net Cash Flow	Net Cash Flow	(*B-A*)	Difference
3	0 (now)	$ (18,000)	$ (60,000)	$ (42,000)	$ (42,000)
4	1	$ (34,400)	$ (25,000)	$ 9,400	$ (32,600)
5	2	$ (34,400)	$ (25,000)	$ 9,400	$ (23,200)
6	3	$ (34,400)	$ (25,000)	$ 9,400	$ (13,800)
7	4	$ (34,400)	$ (34,400)	$ -	$ (13,800)
8	5	$ (34,400)	$ (25,000)	$ 9,400	$ (4,400)
9	6	$ (34,400)	$ (25,000)	$ 9,400	$ 5,000
10	7	$ (34,400)	$ (25,000)	$ 9,400	$ 14,400
11	8	$ (32,400)	$ (17,000)	$ 15,400	$ 29,800
12	Total	$ (291,200)	$ (261,400)		

= – 34400 + 2000

= – 25000 + 8000

= SUM(B3:B11)

We can apply compound interest formulas to "move" cash flows along the cash flow diagram.

Using the standard notation, we find that a present amount, P, can grow into a future amount, F, in N time periods at interest rate i according to the formula below.

$$F = P(1+i)^N$$

In a similar way we can find P given F by

$$P = F(1+i)^{-N}$$

It is common to use standard notation for interest factors.

$$(1+i)^N = (F/P, i, N)$$

This is also known as the *single payment compound amount* factor. The term on the right is read "*F* given *P* at *i*% interest per period for *N* interest periods."

$$(1+i)^{-N} = (P/F, i, N)$$

is called the *single payment present worth* factor.

Finding Economically Equivalent Values

We can use these equations to find economically equivalent values at different points in time.

$2,500 at time zero is equivalent to how much after six years if the interest rate is 8% per year?

$$F = \$2,500(F/P, 8\%, 6) = \$2,500(1.5869) = \$3,967$$

$3,000 at the end of year seven is equivalent to how much today (time zero) if the interest rate is 6% per year?

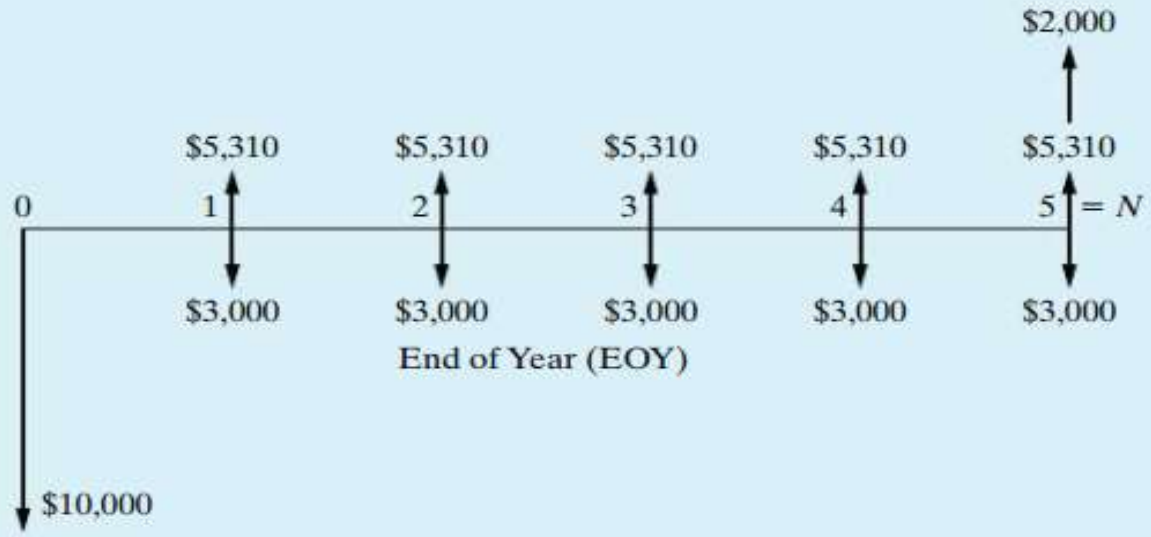
$$P = \$3,000(P/F, 6\%, 7) = \$3,000(0.6651) = \$1,995$$

Cash-Flow Diagramming

Before evaluating the economic merits of a proposed investment, the XYZ Corporation insists that its engineers develop a cash-flow diagram of the proposal. An investment of $10,000 can be made that will produce uniform annual revenue of $5,310 for five years and then have a market (recovery) value of $2,000 at the end of year (EOY) five. Annual expenses will be $3,000 at the end of each year for operating and maintaining the project. Draw a cash-flow diagram for the five-year life of the project. Use the corporation's viewpoint.

Solution

As shown in the figure below, the initial investment of $10,000 and annual expenses of $3,000 are cash outflows, while annual revenues and the market value are cash inflows.



Pause and solve

Betty will need $12,000 in five years to pay for a major overhaul on her tractor engine. She has found an investment that will provide a 5% return on her invested funds. How much does Betty need to invest today so she will have her overhaul funds in five years?

Ans: $9402

P=F(1 + i)^(-N)*

=$12,000(1 + 0.05)^(-5)*

Developing a Net Cash-Flow Table

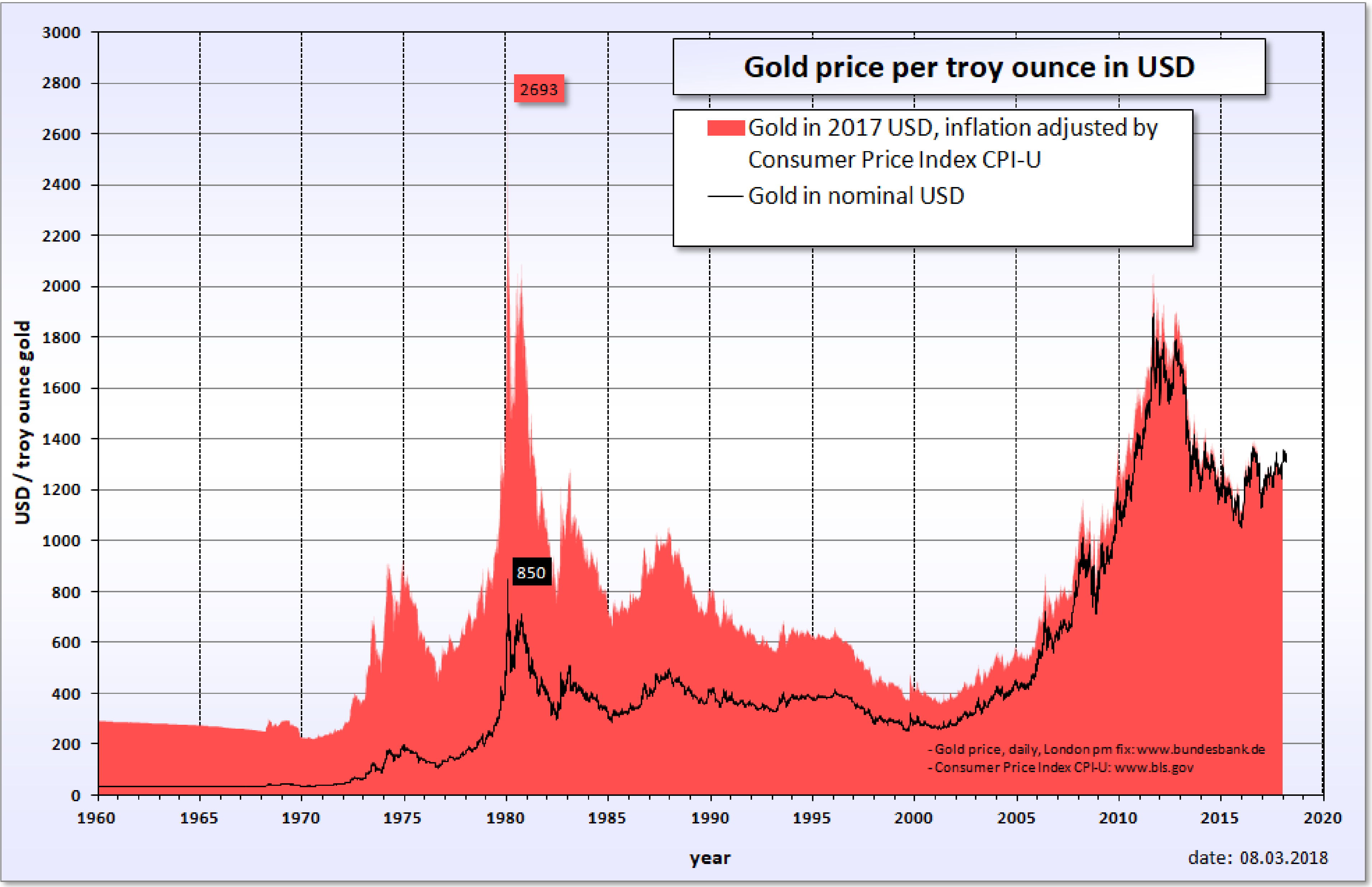
In a company's renovation of a small office building, two feasible alternatives for upgrading the heating, ventilation, and air conditioning (HVAC) system have been identified. Either Alternative *A* *or* Alternative *B* must be implemented. The costs are as follows:

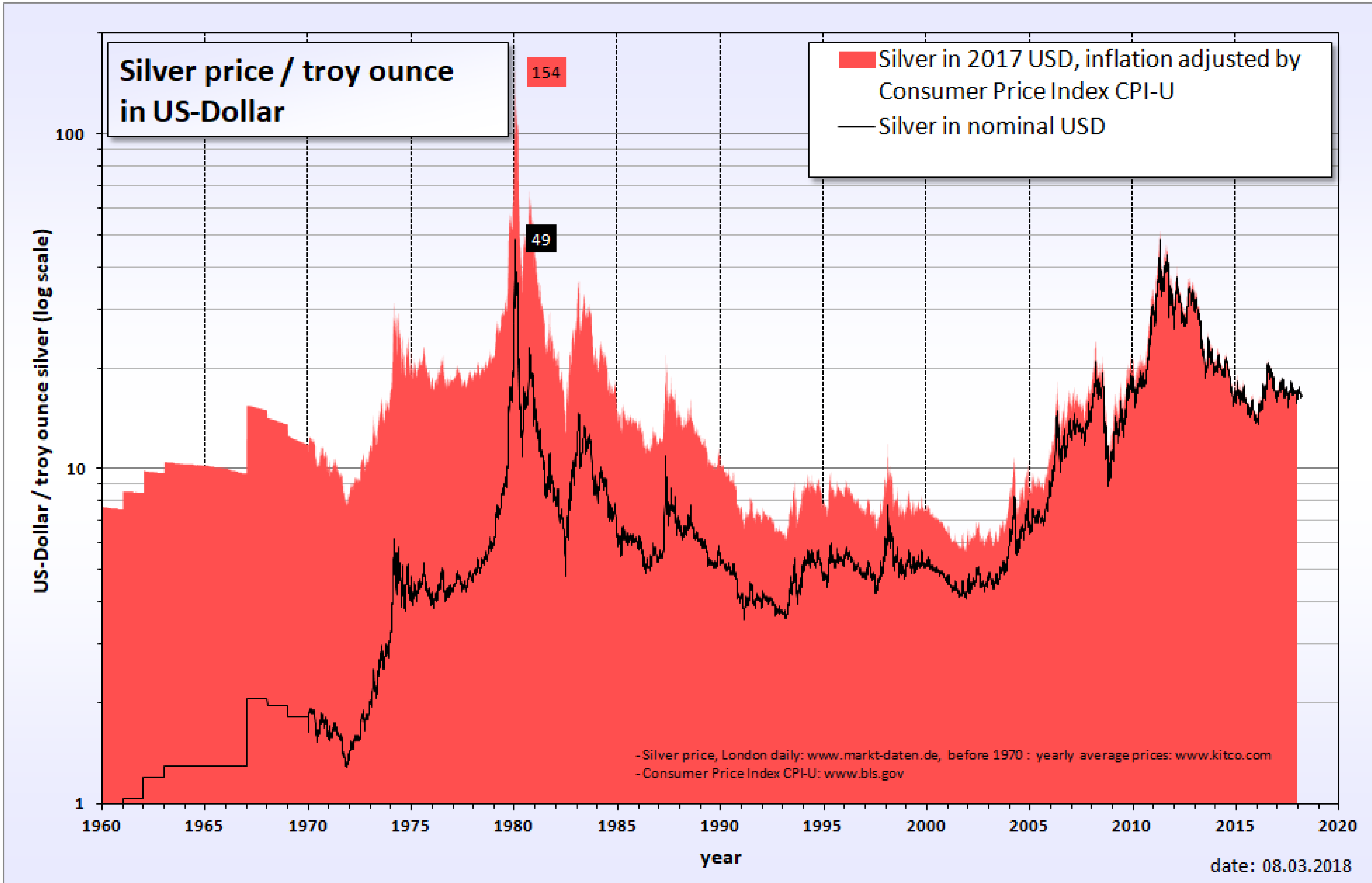
Alternative *A*	*Rebuild (overhaul) the existing HVAC system*	
	• Equipment, labor, and materials to rebuild	$18,000
	• Annual cost of electricity	32,000
	• Annual maintenance expenses	2,400
Alternative *B*	*Install a new HVAC system that utilizes existing ductwork*	
	• Equipment, labor, and materials to install	$60,000
	• Annual cost of electricity	9,000
	• Annual maintenance expenses	16,000
	• Replacement of a major component four years hence ..	9,400

At the end of eight years, the estimated market value for Alternative *A* is $2,000 and for Alternative *B* it is $8,000. Assume that both alternatives will provide comparable service (comfort) over an eight-year period, and assume that the major component replaced in Alternative *B* will have no market value at EOY eight. (1) Use a cash-flow table and end-of-year convention to tabulate the net cash flows for both alternatives. (2) Determine the annual net cash-flow difference between the alternatives ($B - A$).

Solution?

Submit online


Gold price per troy ounce in USD
Gold in 2017 USD, inflation adjusted by Consumer Price Index CPI-U
Gold in nominal USD
2693
850
USD / troy ounce gold
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
year
- Gold price, daily, London pm fix: www.bundesbank.de
- Consumer Price Index CPI-U: www.bls.gov
date: 08.03.2018


Silver price / troy ounce
in US-Dollar
154
49
Silver in 2017 USD, inflation adjusted by Consumer Price Index CPI-U
Silver in nominal USD
US-Dollar / troy ounce silver (log scale)
100
10
1
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
year
-Silver price, London daily: www.markt-daten.de, before 1970 : yearly average prices: www.kitco.com
-Consumer Price Index CPI-U: www.bls.gov
date: 08.03.2018

2000yr Tk15,000/ ~= 2020yr 150,000/-

If we consider gold as reference.

Future Equivalent of a Present Sum

Suppose that you borrow \$8,000 now, promising to repay the loan principal plus accumulated interest in four years at $i = 10\%$ per year. How much would you repay at the end of four years?

Solution

Year	Amount Owed at Start of Year	Interest Owed for Each Year	Amount Owed at End of Year	Total End-of-Year Payment
1	$P = \$8,000$	$iP = \$800$	$P(1+i) = \$8,800$	0
2	$P(1+i) = \$8,800$	$iP(1+i) = \$880$	$P(1+i)^2 = \$9,680$	0
3	$P(1+i)^2 = \$9,680$	$iP(1+i)^2 = \$968$	$P(1+i)^3 = \$10,648$	0
4	$P(1+i)^3 = \$10,648$	$iP(1+i)^3 = \$1,065$	$P(1+i)^4 = \$11,713$	$F = \$11,713$

In general, we see that $F = P(1+i)^N$, and the total amount to be repaid is \$11,713.

The Inflating Price of Gasoline

The average price of gasoline in 2005 was $2.31 per gallon. In 1993, the average price was $1.07.* What was the average annual rate of increase in the price of gasoline over this 12-year period?

Solution

With respect to the year 1993, the year 2005 is in the future. Thus, $P = \$1.07$, $F = \$2.31$, and $N = 12$. Using Equation (4-6), we find $i = \sqrt[12]{2.31/1.07} - 1 = 0.0662$ or 6.62% per year.

* This data was obtained from the Energy Information Administration of the Department of Energy. Historical prices of gasoline and other energy sources can be found at www.eia.doe.gov.

What will happen if we think about gold?

When Will Gasoline Cost $5.00 per Gallon?

In Example 4-5, the average price of gasoline was given as $2.31 in 2005. We computed the average annual rate of increase in the price of gasoline to be 6.62%. If we assume that the price of gasoline will continue to inflate at this rate, how long will it be before we are paying $5.00 per gallon?

Solution

We have $P = \$2.31$, $F = \$5.00$, and $i = 6.62\%$ per year. Using Equation (4-7), we find

$$N = \frac{\log(\$5.00/\$2.31)}{\log(1 + 0.0662)} = \frac{\log(2.1645)}{\log(1.0662)} = 12.05 \text{ years.}$$

So, if gasoline prices continue to increase at the same rate, we can expect to be paying $5.00 per gallon in 2017.

Series of End-of-period Cash Flows.

There are interest factors for a Series of End-of-period Cash Flows. Such a uniform series is also know as *annuity*

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i\%, N)$$

How much will you have in 40 years if you save $3,000 each year and your account earns 8% interest each year?

$$F = \$3,000(F/A, 8\%, 40) = \$3,000(259.0565) = \$777,170$$

Annuity

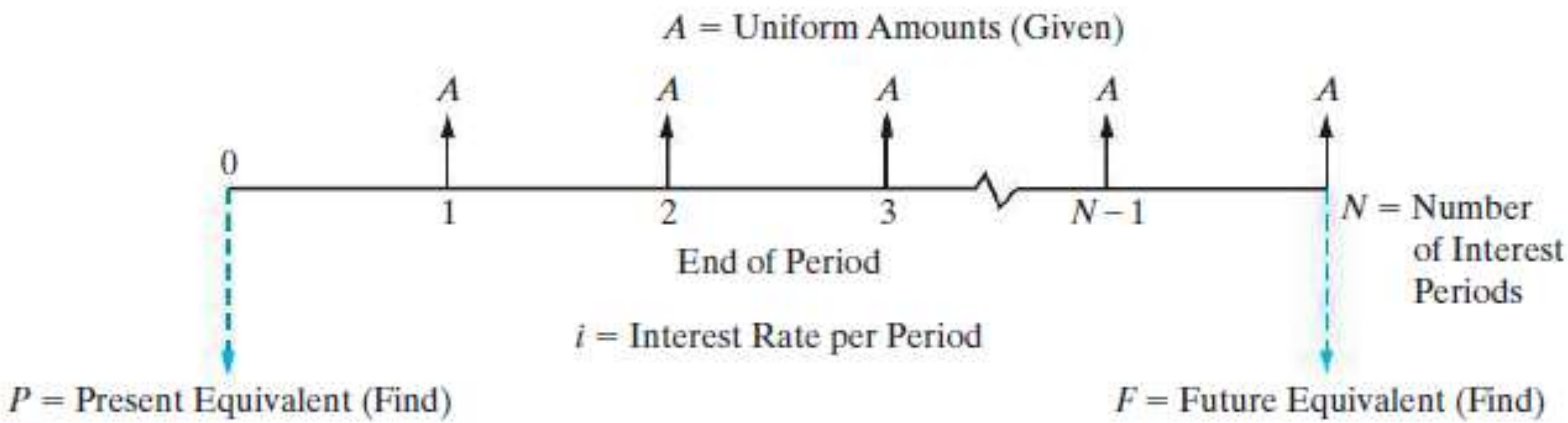


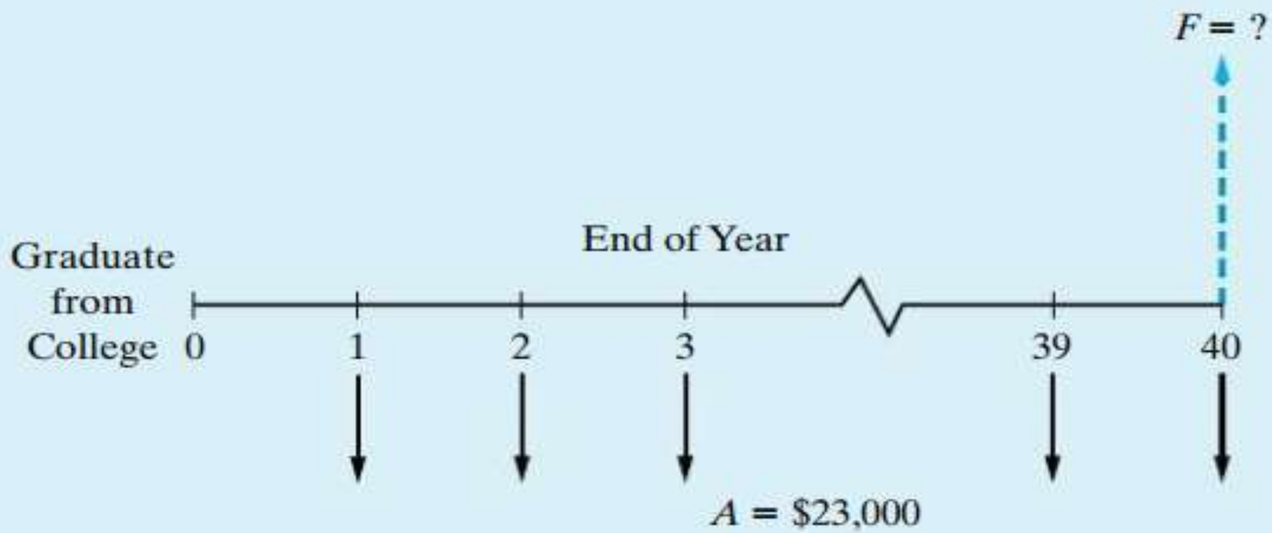
Figure 4-6 General Cash-Flow Diagram Relating Uniform Series (Ordinary Annuity) to Its Present Equivalent and Future Equivalent Values

Future Value of a College Degree

A recent government study reported that a college degree is worth an extra \$23,000 per year in income (A) compared to what a high-school graduate makes. If the interest rate (i) is 6% per year and you work for 40 years (N), what is the future compound amount (F) of this extra income?

Solution

The viewpoint we will use to solve this problem is that of "lending" the \$23,000 of extra annual income to a savings account (or some other investment vehicle). The future equivalent is the amount that can be withdrawn after the 40th deposit is made.



Notice that the future equivalent occurs at the *same time* as the last deposit of $23,000.

$$F = \$23{,}000(F/A,\ 6\%,\ 40)$$
$$= \$23{,}000(154.762)$$
$$= \$3{,}559{,}526$$

The bottom line is "Get your college degree!"

Become a Millionaire by Saving $1.00 a Day!

To illustrate further the amazing effects of compound interest, we consider the credibility of this statement: "If you are 20 years of age and save $1.00 each day for the rest of your life, you can become a millionaire." Let's assume that you live to age 80 and that the annual interest rate is 10% ($i = 10\%$). Under these specific conditions, we compute the future compound amount (F) to be

$$
\begin{aligned}
F &= \$365/\text{year}\ (F/A, 10\%, 60\ \text{years}) \\
&= \$365\ (3{,}034.81) \\
&= \$1{,}107{,}706.
\end{aligned}
$$

Thus, the statement is true for the assumptions given! The moral is to *start saving early* and let the "magic" of compounding work on your behalf!

If you deposit Tk1000/- each month in a scheme of @8% per year, how long will it take to make Tk1.0mill?

Finding the present amount from a series of end-of-period cash flows.

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i\%, N)$$

How much is needed today to provide an annual amount of $50,000 each year for 20 years, at 9% interest each year?

$$P = \$50,000(P/A, 9\%, N) = \$50,000(9.1285) = \$456,427$$

Finding A when given F.

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i\%, N)$$

How much would you need to set aside each year for 25 years, at 10% interest, to have accumulated $1,000,000 at the end of the 25 years?

$$A = \$1,000,000(A/F, 10\%, 25) = \$1,000,000(0.0102) = \$10,200$$

Finding A when given P.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i\%, N)$$

If you had $500,000 today in an account earning 10% each year, how much could you withdraw each year for 25 years?

$$A = \$500,000(A/P, 10\%, 25) = \$500,000(0.1102) = \$55,100$$

Pause and solve

Acme Steamer purchased a new pump for $75,000. They borrowed the money for the pump from their bank at an interest rate of 0.5% per month and will make a total of 24 equal, monthly payments. How much will Acme's monthly payments be?

It can be challenging to solve for *N* or *i*.

- We may know *P*, *A*, and *i* and want to find *N*.
- We may know *P*, *A*, and *N* and want to find *i*.
- These problems present special challenges that are best handled on a spreadsheet.

Finding *N*

Acme borrowed $100,000 from a local bank, which charges them an interest rate of 7% per year. If Acme pays the bank $8,000 per year, how many years will it take to pay off the loan?

$$\$100,000 = \$8,000(P/A, 7\%, N)$$

So,

$$(P/A, 7\%, N) = \frac{\$100,000}{\$8,000} = 12.5 = \frac{(1.07)^N - 1}{0.07(1.07)^N}$$

This can be solved by using the interest tables and interpolation, but we generally resort to a computer solution.

Finding *i*

Jill invested $1,000 each year for five years in a local company and sold her interest after five years for $8,000. What annual rate of return did Jill earn?

$$\$8,000 = \$1,000(F/A, i\%, 5)$$

So,

$$(F/A, i\%, 5) = \frac{\$8,000}{\$1,000} = 8 = \frac{(1+i)^5 - 1}{i}$$

Again, this can be solved using the interest tables and interpolation, but we generally resort to a computer solution.

Spreadsheet functions to find *N* and *i*.

The Excel function used to solve for *N* is

NPER(*rate*, *pmt*, *pv*), which will compute the number of payments of magnitude *pmt* required to pay off a present amount (*pv)* at a fixed interest rate (*rate*).

https://exceljet.net/excel-functions/excel-nper-function

One Excel function used to solve for *i* is

RATE(*nper*, *pmt*, *pv*, *fv*), which returns a fixed interest rate for an annuity of *pmt* that lasts for *nper* periods to either its present value (*pv*) or future value (*fv*).

Cash flows that do not occur until some time in the future.

- Deferred annuities are uniform series that do not begin until some time in the future.
- If the annuity is deferred J periods then the first payment (cash flow) begins at the end of period $J+1$.

Finding the value at time *0* of a deferred annuity is a two-step process.

1. Use (*P/A*, *i*%, *N*-*J*) find the value of the deferred annuity at the end of period *J* (where there are *N*-*J* cash flows in the annuity).
2. Use (*P/F*, *i*%, *J*) to find the value of the deferred annuity at time zero.

$$P_0 = A(P/A, i\%, N-J)(P/F, i\%, J)$$

Pause and solve

Irene just purchased a new sports car and wants to also set aside cash for future maintenance expenses. The car has a bumper-to-bumper warranty for the first five years. Irene estimates that she will need approximately $2,000 per year in maintenance expenses for years 6-10, at which time she will sell the vehicle. How much money should Irene deposit into an account today, at 8% per year, so that she will have sufficient funds in that account to cover her projected maintenance expenses?

Sometimes cash flows change by a constant amount each period.

We can model these situations as *a uniform gradient* of cash flows. The table below shows such a gradient.

End of Period	Cash Flows
1	0
2	G
3	$2G$
:	:
N	$(N-1)G$

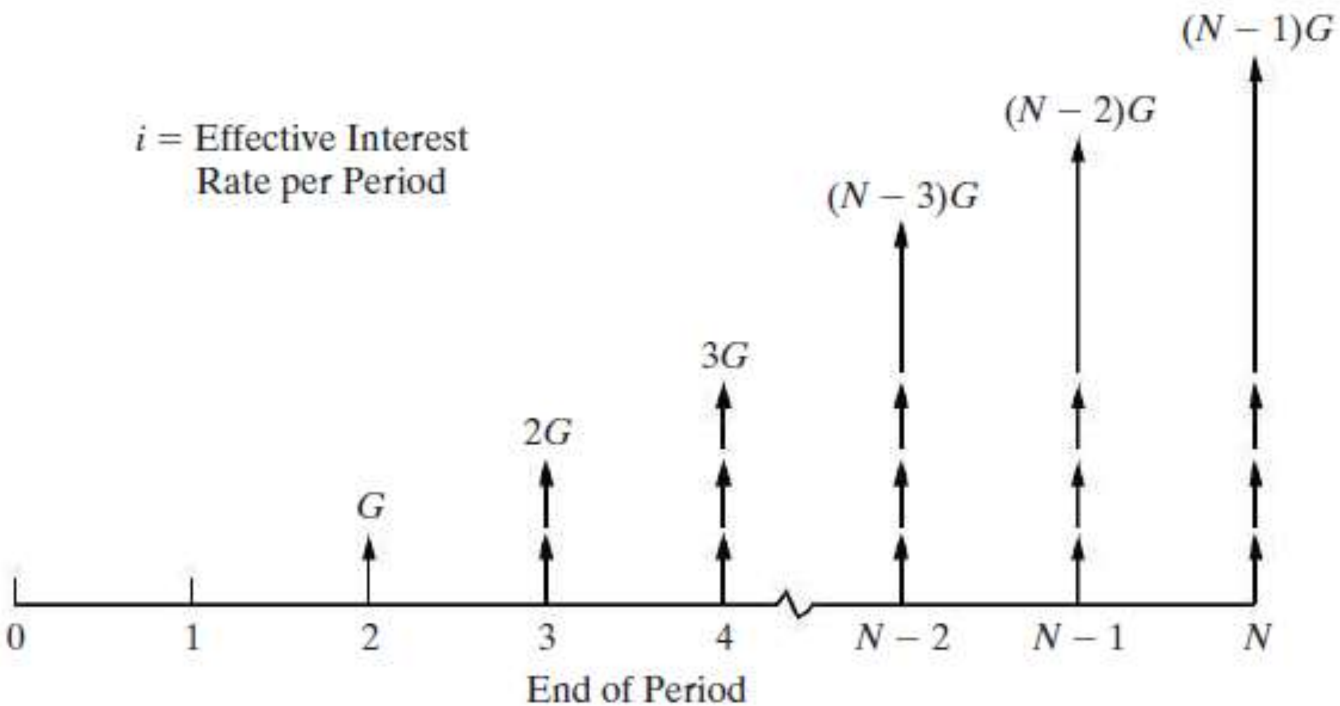


Figure 4-13 Cash-Flow Diagram for a Uniform Gradient Increasing by *G* Dollars per Period

It is easy to find the present value of a uniform gradient series.

Similar to the other types of cash flows, there is a formula (albeit quite complicated) we can use to find the present value, and a set of factors developed for interest tables.

$$(P/G, i\%, N) = \frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]$$

We can also find *A* or *F* equivalent to a *uniform gradient* series.

$$(A/G, i\%, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$$

$$(F/G, i\%, N) = \frac{1}{i}(F/A, i\%, N) - \frac{N}{i}$$

The annual equivalent of this series of cash flows can be found by considering an annuity portion of the cash flows and a gradient portion.

End of Year	Cash Flows ($)
1	2,000
2	3,000
3	4,000
4	5,000

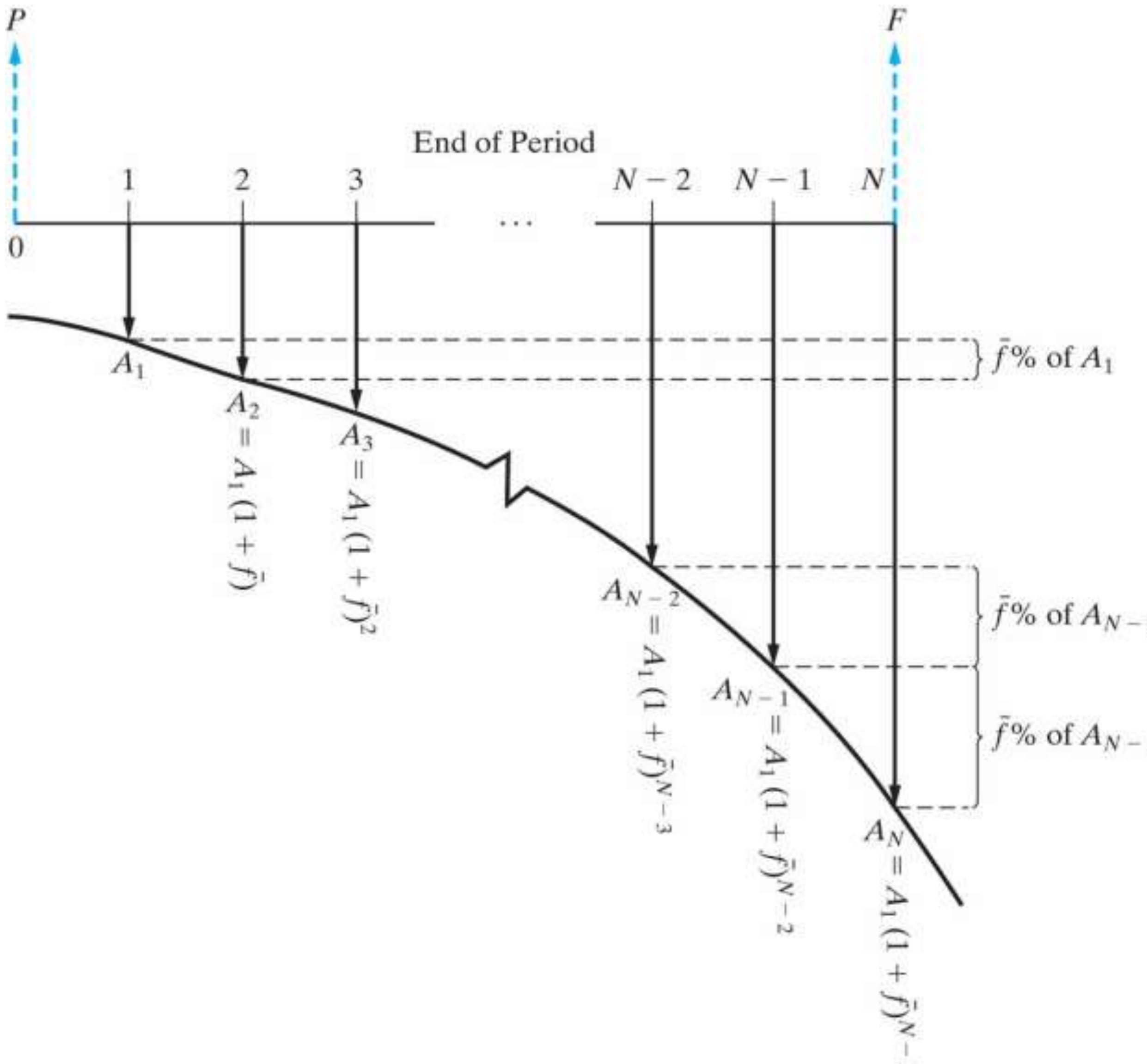
End of Year	Annuity ($)	Gradient ($)
1	2,000	0
2	2,000	1,000
3	2,000	2,000
4	2,000	3,000

$$A = \$2,000 + \$1,000(A/G, 8\%, 4) = \$3,404$$

Sometimes cash flows change by a constant rate, $\bar{f}$, each period--this is a *geometric gradient series*.

This table presents a geometric gradient series. It begins at the end of year 1 and has a rate of growth, $\bar{f}$, of 20%.

End of Year	Cash Flows ($)
1	1,000
2	1,200
3	1,440
4	1,728


P
F
End of Period
0
1
2
3
. . .
N − 2
N − 1
N
A_1
$A_2 = A_1(1 + \bar{f})$
$A_3 = A_1(1 + \bar{f})^2$
$A_{N-2} = A_1(1 + \bar{f})^{N-3}$
$A_{N-1} = A_1(1 + \bar{f})^{N-2}$
$A_N = A_1(1 + \bar{f})^{N-1}$
$\bar{f}$% of A_1
$\bar{f}$% of A_{N-2}
$\bar{f}$% of A_{N-1}

We can find the **present value** of a geometric series by using the appropriate formula below.

If $\bar{f} \neq i$

$$\frac{A_1[1-(P/F,i\%,N)(F/P,\bar{f}\%,N]}{1-\bar{f}}$$

If $\bar{f} = i$

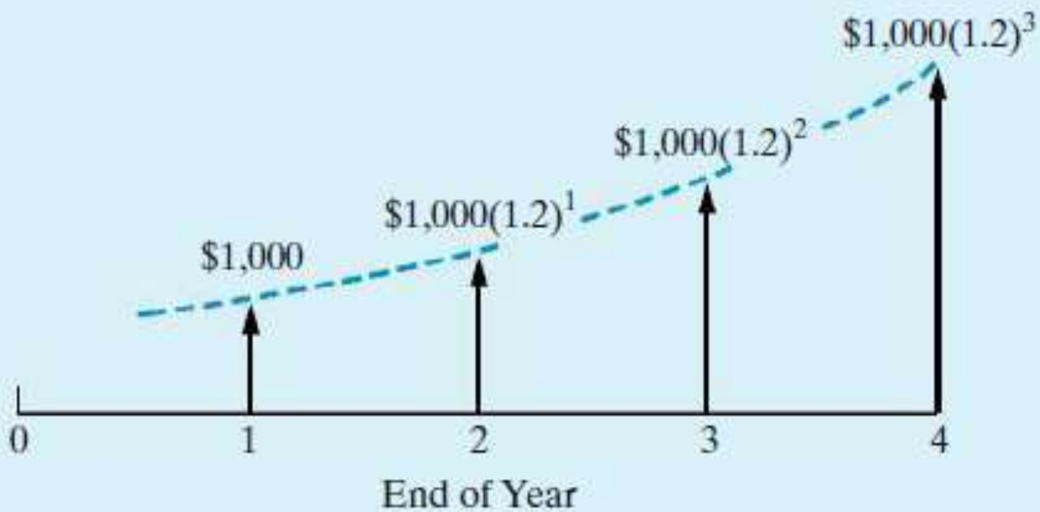
$$A_1 N(P/F,i\%,1)$$

Where A_1 is the initial cash flow in the series.

Equivalence Calculations for an Increasing Geometric Gradient Series

Consider the following EOY geometric sequence of cash flows and determine the *P*, *A*, and *F* equivalent values. The rate of increase is 20% per year after the first year, and the interest rate is 25% per year.

Solution



$$P = \frac{\$1{,}000\,[1 - (P/F, 25\%, 4)(F/P, 20\%, 4)]}{0.25 - 0.20}$$

Pause and solve

Acme Miracle projects good things for their new weight loss pill, LoseIt. Revenues this year are expected to be $1.1 million, and Acme believes they will increase 15% per year for the next 5 years. What are the present value and equivalent annual amount for the anticipated revenues? Acme uses an interest rate of 20%.

When interest rates vary with time different procedures are necessary.

- Interest rates often change with time (e.g., a variable rate mortgage).
- We often must resort to moving cash flows one period at a time, reflecting the interest rate for that single period.

The present equivalent of a cash flow occurring at the end of period N can be computed with the equation below, where i_k is the interest rate for the k^{th} period.

$$P = \frac{F_N}{\prod_{k=1}^{N}(1+i_k)}$$

If F_4 = \$2,500 and i_1=8%, i_2=10%, and i_3=11%, then

$$P = \$2,500(P/F, 8\%, 1)(P/F, 10\%, 1)(P/F, 11\%, 1)$$

$$P = \$2,500(0.9259)(0.9091)(0.9009) = \$1,896$$

Nominal and effective interest rates.

- More often than not, the time between successive compounding, or the interest period, is less than one year (e.g., daily, monthly, quarterly).
- The annual rate is known as a *nominal* rate.
- A *nominal* rate of 12%, compounded monthly, means an interest of 1% (12%/12) would accrue each month, and the annual rate would be *effectively* somewhat greater than 12%.
- The more frequent the compounding the greater the *effective* interest.

The effect of more frequent compounding can be easily determined.

Let *r* be the nominal, annual interest rate and *M* the number of compounding periods per year. We can find, *i*, the effective interest by using the formula below.

$$i = (1 + \frac{r}{M})^M - 1$$

Finding effective interest rates.

For an 18% nominal rate, compounded quarterly, the effective interest is.

$$i = (1 + \frac{0.18}{4})^4 - 1 = 19.25\%$$

For a 7% nominal rate, compounded monthly, the effective interest is.

$$i = (1 + \frac{0.07}{12})^{12} - 1 = 7.23\%$$

Interest can be compounded continuously.

- Interest is typically compounded at the end of discrete periods.
- In most companies cash is always flowing, and should be immediately put to use.
- We can allow compounding to occur continuously throughout the period.
- The effect of this compared to discrete compounding is small in most cases.

Effective interest formula to derive the interest factors.

$$i = (1 + \frac{r}{M})^M - 1$$

As the number of compounding periods gets larger (*M* gets larger), we find that

$$i = e^r - 1$$

Continuous compounding interest factors.

$$(P/F, \underline{r}\%, N) = e^{-rN}$$

$$(F/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^{r} - 1}$$

$$(P/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^{r} - 1)}$$

The other factors can be found from these.

Prepaying a Loan—Finding *N*

Your company has a $100,000 loan for a new security system it just bought. The annual payment is $8,880 and the interest rate is 8% per year for 30 years. Your company decides that it can afford to pay $10,000 per year. After how many payments (years) will the loan be paid off?

A Retirement Savings Plan

On your 23rd birthday you decide to invest $4,500 (10% of your annual salary) in a mutual fund earning 7% per year. You will continue to make annual deposits equal to 10% of your annual salary until you retire at age 62 (40 years after you started your job). You expect your salary to increase by an average of 4% each year during this time. How much money will you have accumulated in your mutual fund when you retire?

Compounding with Changing Interest Rates

Ashea Smith is a 22-year-old senior who used the Stafford loan program to borrow \$4,000 four years ago when the interest rate was 4.06% per year. \$5,000 was borrowed three years ago at 3.42%. Two years ago she borrowed \$6,000 at 5.23%, and last year \$7,000 was borrowed at 6.03% per year. Now she would like to consolidate her debt into a single 20-year loan with a 5% fixed annual interest rate. If Ashea makes annual payments (starting in one year) to repay her total debt, what is the amount of each payment?

TABLE 4-2 Discrete Cash-Flow Examples Illustrating Equivalence

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

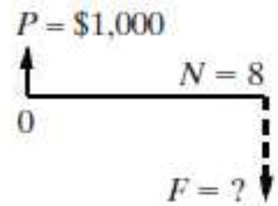
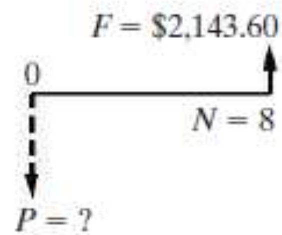
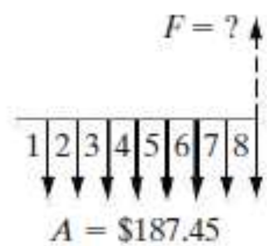
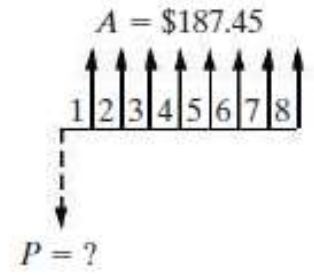
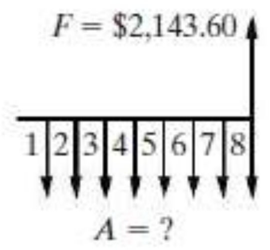
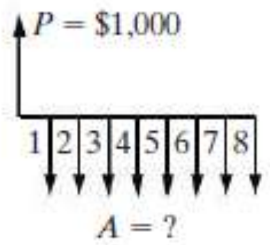
To Find:	Given:	(a) In Borrowing–Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram[a]	Solution
For single cash flows:					
F	P	A firm borrows \$1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year?	What is the future equivalent at the end of eight years of \$1,000 at the beginning of those eight years?		$F = P(F/P, 10\%, 8)$ $= \$1{,}000(2.1436)$ $= \$2{,}143.60$
P	F	A firm wishes to have \$2,143.60 eight years from now. What amount should be deposited now to provide for it?	What is the present equivalent of \$2,143.60 received eight years from now?		$P = F(P/F, 10\%, 8)$ $= \$2{,}143.60(0.4665)$ $= \$1{,}000.00$
For uniform series:					
F	A	If eight annual deposits of \$187.45 each are placed in an account, how much money has accumulated immediately after the last deposit?	What amount at the end of the eighth year is equivalent to eight EOY payments of \$187.45 each?		$F = A(F/A, 10\%, 8)$ $= \$187.45(11.4359)$ $= \$2{,}143.60$

TABLE 4-2 *(Continued)*

P	A	How much should be deposited in a fund now to provide for eight EOY withdrawals of \$187.45 each?	What is the present equivalent of eight EOY payments of \$187.45 each?		$P = A(P/A, 10\%, 8)$ $= \$187.45(5.3349)$ $= \$1{,}000.00$
A	F	What uniform annual amount should be deposited each year in order to accumulate \$2,143.60 at the time of the eighth annual deposit?	What uniform payment at the end of eight successive years is equivalent to \$2,143.60 at the end of the eighth year?		$A = F(A/F, 10\%, 8)$ $= \$2{,}143.60(0.0874)$ $= \$187.45$
A	P	What is the size of eight equal annual payments to repay a loan of \$1,000? The first payment is due one year after receiving the loan.	What uniform payment at the end of eight successive years is equivalent to \$1,000 at the beginning of the first year?		$A = P(A/P, 10\%, 8)$ $= \$1{,}000(0.18745)$ $= \$187.45$

[a] The cash-flow diagram represents the example as stated in borrowing-lending terminology.

TABLE 4-3 Discrete Compounding-Interest Factors and Symbols[a]

To Find:	Given:	Factor by which to Multiply "Given"[a]	Factor Name	Factor Functional Symbol[b]
For single cash flows:				
F	P	$(1+i)^N$	Single payment compound amount	$(F/P, i\%, N)$
P	F	$\frac{1}{(1+i)^N}$	Single payment present worth	$(P/F, i\%, N)$
For uniform series (annuities):				
F	A	$\frac{(1+i)^N-1}{i}$	Uniform series compound amount	$(F/A, i\%, N)$
P	A	$\frac{(1+i)^N-1}{i(1+i)^N}$	Uniform series present worth	$(P/A, i\%, N)$
A	F	$\frac{i}{(1+i)^N-1}$	Sinking fund	$(A/F, i\%, N)$
A	P	$\frac{i(1+i)^N}{(1+i)^N-1}$	Capital recovery	$(A/P, i\%, N)$

[a] i equals effective interest rate per interest period; N, number of interest periods; A, uniform series amount (occurs at the end of each interest period); F, future equivalent; P, present equivalent.

[b] The functional symbol system is used throughout this book.