Nonparametric Methods: Goodness of Fit Tests

Chapter 17

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Learning Objectives

LO1 Conduct a test of hypothesis comparing an observed set of frequencies to an expected distribution.

LO2 List and explain the characteristics of the *chi-square distribution*.

LO3 Conduct a goodness-of-fit test for unequal expected frequencies.

LO4 Conduct a test of hypothesis to verify that data grouped into a frequency distribution is a sample from a normal distribution.

LO5 Use graphical methods to determine if a set of sample data is from a normal distribution.

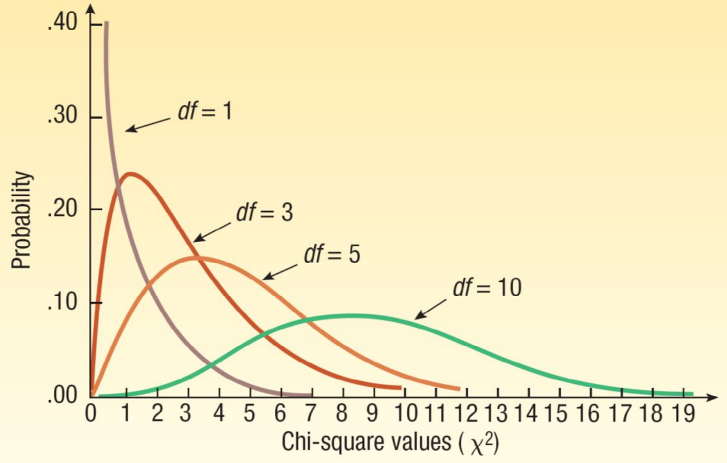
LO6 Conduct a test of hypothesis to determine whether two classification criteria are related.

LO1 List and explain the characteristics of the *chi-square distribution*.

Characteristics of the Chi-Square Distribution

The major characteristics of the chi-square distribution

- It is positively skewed.
- It is non-negative.
- It is based on degrees of freedom.
- When the degrees of freedom change a new distribution is created.



A Portion of the Chi-Square Table

Degrees of Freedom *df*	Right-Tail Area .10	.05	.02	.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086

LO2 Conduct a test of hypothesis comparing an observed set of frequencies to an expected distribution.

Goodness-of-Fit Test: Equal Expected Frequencies

- Let f_0 and f_e be the observed and expected frequencies respectively.
- H_0: There is no difference between the observed and the expected frequencies
- H_1: There is a difference between the observed and the expected frequencies.
- The test statistic is:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

- The critical value is a chi-square value with (k-1) degrees of freedom, where k is the number of categories

EXAMPLE

Ms. Jan Kilpatrick is the marketing manager for a manufacturer of sports cards. She plans to begin selling a series of cards with pictures and playing statistics of former Major League Baseball players. One of the problems is the selection of the former players. At a baseball card show at Southwyck Mall last weekend, she set up a booth and offered cards of the following six Hall of Fame baseball players: Tom Seaver, Nolan Ryan, Ty Cobb, George Brett, Hank Aaron, and Johnny Bench. At the end of the day she sold a total of 120 cards. The number of cards sold for each old-time player is shown in the table on the right. **Can she conclude the sales are not the same for each player? Use 0.05 significance level.**

Player	Cards Sold
Tom Seaver	13
Nolan Ryan	33
Ty Cobb	14
George Brett	7
Hank Aaron	36
Johnny Bench	17
Total	120

LO2

Goodness-of-Fit Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: there is no difference between f_o and f_e
H_1: there is a difference between f_o and f_e

Step 2: Select the level of significance.

α = 0.05 as stated in the problem

Step 3: Select the test statistic.

The test statistic follows the chi-square distribution, designated as χ^2

Step 4: Formulate the decision rule.

$$\text{Reject } H_0 \text{ if } \chi^2 > \chi^2_{\alpha,k-1}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > \chi^2_{\alpha,k-1}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > \chi^2_{.05,5}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > 11.070$$

Step 5: Compute the value of the chi-square statistic and make a decision

$$\chi^2 = \sum\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

Baseball Player	f_o	f_e	(1) $(f_o - f_e)$	(2) $(f_o - f_e)^2$	(3) $\frac{(f_o - f_e)^2}{f_e}$
Tom Seaver	13	20	−7	49	49/20 = 2.45
Nolan Ryan	33	20	13	169	169/20 = 8.45
Ty Cobb	14	20	−6	36	36/20 = 1.80
George Brett	7	20	−13	169	169/20 = 8.45
Hank Aaron	36	20	16	256	256/20 = 12.80
Johnny Bench	17	20	−3	9	9/20 = 0.45
		Must be →	0		χ^2 → 34.40

The computed χ^2 of 34.40 larger than the critical value of 11.070. The decision, therefore, is to reject H_0 at the .05 level .

Conclusion: The difference between the observed and the expected frequencies is not due to chance. Rather, the differences between f_0 and f_e and are large enough to be considered significant. It is unlikely that card sales are the same among the six players.

LO3 Conduct a goodness-of-fit test for unequal expected frequencies.

Goodness-of-Fit Test: Unequal Expected Frequencies

- Let f_0 and f_e be the observed and expected frequencies respectively.

The Hypothesis

- H_0: There is no difference between the observed and expected frequencies.
- H_1: There is a difference between the observed and the expected frequencies.

The test statistic is computed using the following formula:

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

EXAMPLE

The American Hospital Administrators Association (AHAA) reports the following information concerning the number of times senior citizens are admitted to a hospital during a one-year period. Forty percent are not admitted; 30 percent are admitted once; 20 percent are admitted twice, and the remaining 10 percent are admitted three or more times.

A survey of 150 residents of Bartow Estates, a community devoted to active seniors located in central Florida, revealed 55 residents were not admitted during the last year, 50 were admitted to a hospital once, 32 were admitted twice, and the rest of those in the survey were admitted three or more times.

Can we conclude the survey at Bartow Estates is consistent with the information suggested by the AHAA? Use the .05 significance level.

Goodness-of-Fit Test: Unequal Expected Frequencies - Example

LO3

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: There is no difference between local and national experience for hospital admissions.

H_1: There is a difference between local and national experience for hospital admissions.

Step 2: Select the level of significance.

α = 0.05 as stated in the problem

Step 3: Select the test statistic.

The test statistic follows the chi-square distribution, designated as χ^2

Step 4: Formulate the decision rule.

Reject H_0 if $\chi^2 > \chi^2_{\alpha, k-1}$

$$\sum \left[\frac{(f_o - f_e)^2}{f_e} \right] > \chi^2_{\alpha, k-1}$$

$$\sum \left[\frac{(f_o - f_e)^2}{f_e} \right] > \chi^2_{.05, 4-1}$$

$$\sum \left[\frac{(f_o - f_e)^2}{f_e} \right] > \chi^2_{.05, 3}$$

$$\sum \left[\frac{(f_o - f_e)^2}{f_e} \right] > 7.815$$

Goodness-of-Fit Test: Unequal Expected Frequencies - Example

LO3

Step 5: Compute the statistic and make a decision

$$\chi^2 = \sum\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

Number of Times Admitted	(f_o)	(f_e)	$f_o - f_e$	$(f_o - f_e)^2/f_e$
0	55	60	−5	0.4167
1	50	45	5	0.5556
2	32	30	2	0.1333
3 or more	13	15	−2	0.2667
Total	150	150	0	1.3723

The computed χ^2 of 1.3723 is NOT greater than the critical value of 7.815 – we cannot reject the null hypothesis. The difference between the observed and the expected frequencies is due to chance.

We conclude that there is no evidence of a difference between the local and national experience for hospital admissions.

LO6 Conduct a test of hypothesis to determine whether two classification criteria are related.

Contingency Table Analysis

A contingency table is used to investigate whether two traits or characteristics are related. Each observation is classified according to two criteria. We use the usual hypothesis testing procedure.

- The degrees of freedom is equal to:

  (number of rows-1)(number of columns-1).

The expected frequency is computed as: $f_e = \frac{\text{(Row total)(Column total)}}{\text{Grand total}}$

We can use the chi-square statistic to formally test for a relationship between two nominal-scaled variables. To put it another way, Is one variable *independent* of the other?

- Ford Motor Company operates an assembly plant in Dearborn, Michigan. The plant operates three shifts per day, 5 days a week. The quality control manager wishes to compare the quality level on the three shifts. Vehicles are classified by quality level (acceptable, unacceptable) and shift (day, afternoon, night). Is there a difference in the quality level on the three shifts? That is, is the quality of the product related to the shift when it was manufactured? Or is the quality of the product independent of the shift on which it was manufactured?

- A sample of 100 drivers who were stopped for speeding violations was classified by gender and whether or not they were wearing a seat belt. For this sample, is wearing a seatbelt related to gender?

- Does a male released from federal prison make a different adjustment to civilian life if he returns to his hometown or if he goes elsewhere to live? The two variables are adjustment to civilian life and place of residence. Note that both variables are measured on the nominal scale.

Contingency Analysis - Example

The Federal Correction Agency is investigating the "Does a male released from federal prison make a different adjustment to civilian life if he returns to his hometown or if he goes elsewhere to live?" To put it another way, is there a relationship between adjustment to civilian life and place of residence after release from prison? Use the .01 significance level.

The agency's psychologists interviewed 200 randomly selected former prisoners. Using a series of questions, the psychologists classified the adjustment of each individual to civilian life as outstanding, good, fair, or unsatisfactory.

The classifications for the 200 former prisoners were tallied as follows. Joseph Camden, for example, returned to his hometown and has shown outstanding adjustment to civilian life. His case is one of the 27 tallies in the upper left box (circled).

Residence after Release from Prison	Adjustment to Civilian Life			
	Outstanding	Good	Fair	Unsatisfactory
Hometown	卌 卌 卌 卌 卌 ||	卌 卌 卌 卌 卌 卌 卌	卌 卌 卌 卌 卌 卌 |||	卌 卌 卌 卌 卌
Not hometown	卌 卌 |||	卌 卌 卌	卌 卌 卌 卌 卌 ||	卌 卌 卌 卌 卌

Residence after Release from Prison	Adjustment to Civilian Life				
	Outstanding	Good	Fair	Unsatisfactory	Total
Hometown	27	35	33	25	120
Not hometown	13	15	27	25	80
Total	40	50	60	50	200

LO6

Contingency Analysis - Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: There is no relationship between adjustment to civilian life and where the individual lives after being released from prison.

H_1: There is a relationship between adjustment to civilian life and where the individual lives after being released from prison.

Step 2: Select the level of significance.

α = 0.01 as stated in the problem

Step 3: Select the test statistic.

The test statistic follows the chi-square distribution, designated as χ^2

Step 4: Formulate the decision rule.

$$\text{Reject } H_0 \text{ if } \chi^2 > \chi^2_{\alpha,(r-1)(c-1)}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > \chi^2_{\alpha,(2-1)(4-1)}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > \chi^2_{.01,(1)(3)}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > \chi^2_{.01,3}$$

$$\sum\left[\frac{(f_o - f_e)^2}{f_e}\right] > 11.345$$

LO6

Computing Expected Frequencies (f_e)

EXPECTED FREQUENCY $$f_e = \frac{\text{(Row total)(Column total)}}{\text{Grand total}}$$

$\frac{(120)(50)}{200}$

Residence after Release from Prison	Outstanding		Good		Fair		Unsatisfactory		Total	
	f_o	f_e	f_o	f_e	f_o	f_e	f_o	f_e	f_o	f_e
Hometown	27	24	35	30	33	36	25	30	120	120
Not hometown	13	16	15	20	27	24	25	20	80	80
Total	40	40	50	50	60	60	50	50	200	200

Adjustment to Civilian Life

Must be equal (Outstanding totals f_o and f_e)

$\frac{(80)(50)}{200}$

Must be equal (Total f_o and f_e)

Computing the Chi-square Statistic

Residence after Release from Prison	Adjustment to Civilian Life: Outstanding f_o	Outstanding f_e	Good f_o	Good f_e	Fair f_o	Fair f_e	Unsatisfactory f_o	Unsatisfactory f_e	Total f_o	Total f_e
Hometown	27	24	35	30	33	36	25	30	120	120
Not hometown	13	16	15	20	27	24	25	20	80	80
Total	40	40	50	50	60	60	50	50	200	200

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

Starting with the upper left cell:

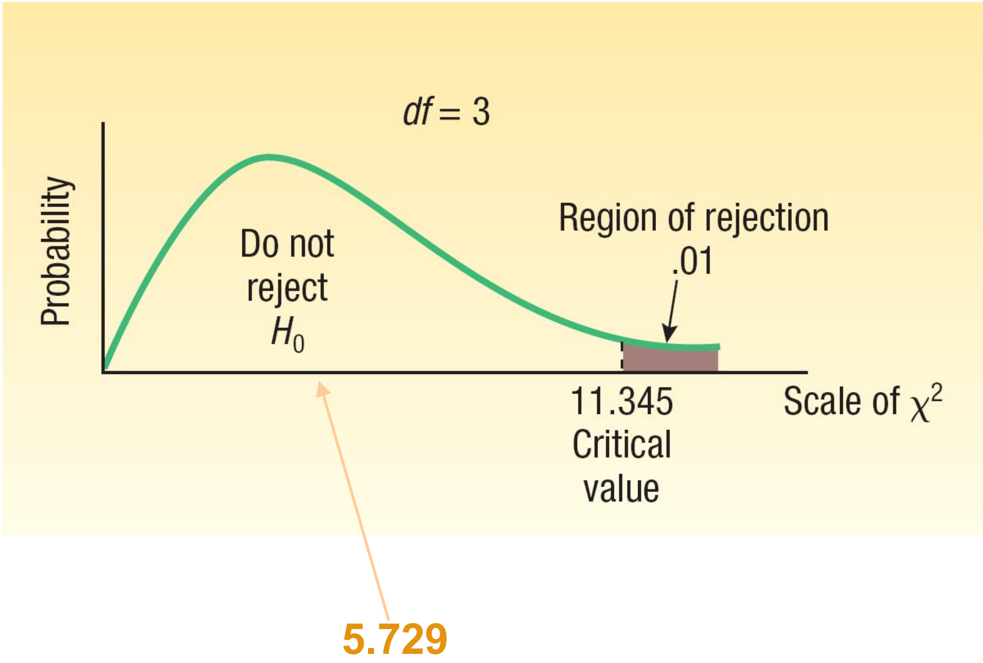
$$\chi^2 = \frac{(27-24)^2}{24} + \frac{(35-30)^2}{30} + \frac{(33-36)^2}{36} + \frac{(25-30)^2}{30} + \frac{(13-16)^2}{16} + \frac{(15-20)^2}{20} + \frac{(27-24)^2}{24} + \frac{(25-20)^2}{20}$$

$$= 0.375 + 0.833 + 0.250 + 0.833 + 0.563 + 1.250 + 0.375 + 1.250$$

$$= 5.729$$

LO6

Conclusion



The computed χ^2 of 5.729 is in the "Do not rejection H_0" region. The null hypothesis is not rejected at the .01 significance level.

We conclude there is no evidence of a relationship between adjustment to civilian life and where the prisoner resides after being released from prison. For the Federal Correction Agency's advisement program, adjustment to civilian life is not related to where the ex-prisoner lives.

LO6

Contingency Analysis - Minitab