ME265: Thermal Engineering & Heat Transfer

4.3.4 Natural Convection

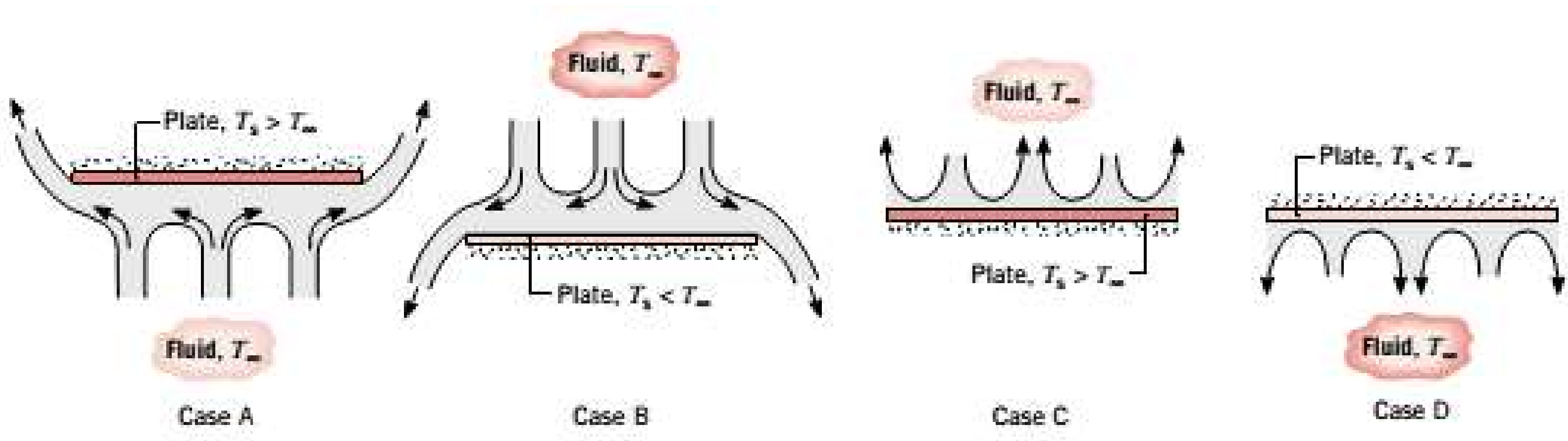


Figure: Free convection buoyancy-driven flows for hot ($T_s > T_\infty$) and cold ($T_s < T_\infty$) horizontal plates:

Case A — hot surface facing downwards,
Case B — cold surface facing upwards,
Case C — hot surface facing upwards, and
Case D — cold surface facing downwards.

Reference: Moran et al. (2003) Introduction to Thermal systems Engineering, © John Wiley and Sons Inc. p.443

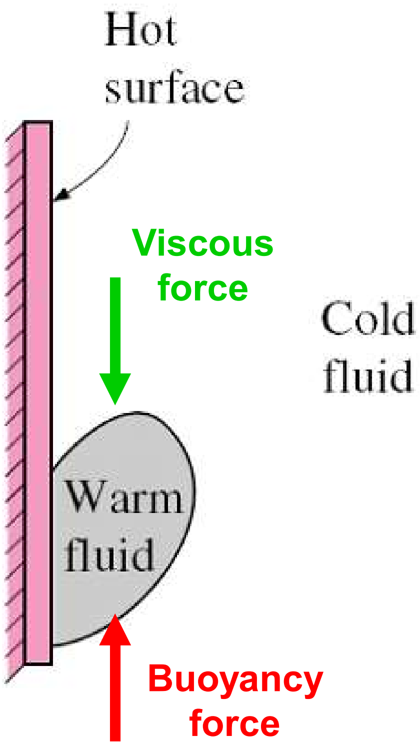
4.3.4 Natural Convection

❑ Section objectives:

- Understand the **physical mechanism** of natural convection and **significant dimensionless number**.
- Estimate natural convection coefficient using **empirical correlations**
- Examine natural convection from finned surfaces, and **determining the optimum fin spacing**.
- Analyze **natural convection inside enclosures** such as double-pane windows.

4.3.4 Natural Convection

❑ Grashof Number:



$$\textbf{Grashof Number} = \frac{\textbf{Buoyancy Force}}{\textbf{Viscous Force}}$$

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2}$$

- g gravitational acceleration, m/s^2
- β coefficient of volume expansion, 1/K
- (β = 1/T for ideal gases)
- T_s temperature of the surface, °C
- T_∞ temperature of the fluid sufficiently far from the surface, °C
- L_c characteristic length of the geometry, m
- ν kinematic viscosity of the fluid, m^2/s

- The flow regime in natural convection is governed by the *Grashof number*
- $Gr_L > 10^9$ flow is turbulent

4.3.4 Natural Convection

- Natural convection heat transfer on a surface depends on:
  (i) geometry,
  (ii) orientation,
  (iii) variation of temperature on the surface, and
  (iv) thermophysical properties of the fluid.

- The simple empirical correlations for the average *Nusselt number* in natural convection are of the form

$$Nu = \frac{hL_c}{k} = C \cdot \left(Gr_L \cdot \Pr\right)^n = C \cdot Ra_L^n \quad \text{…… (3.4.6)}$$

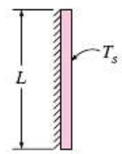
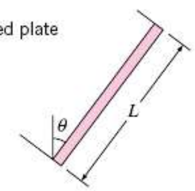
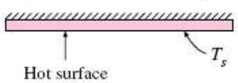
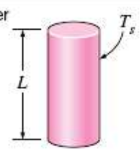
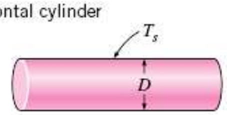
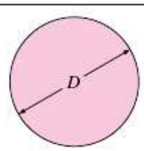
- Where Ra_L is the **Rayleigh number** $Ra_L = Gr_L \cdot \Pr = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2}\Pr$

- The values of the constants **C and n** depend on the geometry of the surface and the flow regime
- The value of **n** is usually **1/4 for laminar flow** and **1/3 for turbulent flow**. The value of the constant **C is normally less than 1**.

4.3.4 Natural Convection

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{10}–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ $\text{Nu} = 0.27\text{Ra}_L^{1/4}$
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$

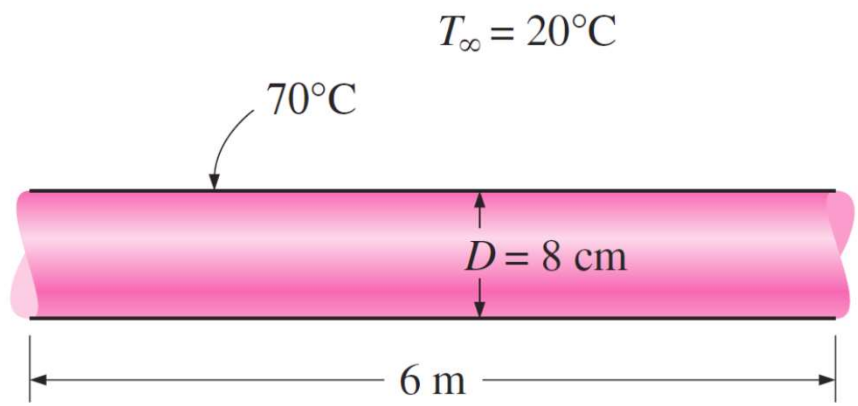
All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_\infty)/2$.

The relations for the constant surface temperature and constant surface heat flux cases are nearly identical.

4.3.4 Natural Convection

EP# 3.11

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.



Assumptions:

1. Steady operating conditions exist.
2. Air is an ideal gas
3. The local atmospheric pressure is 1 atm.

Properties

The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$k = 0.02699$ W/m°C   Pr = 0.7241

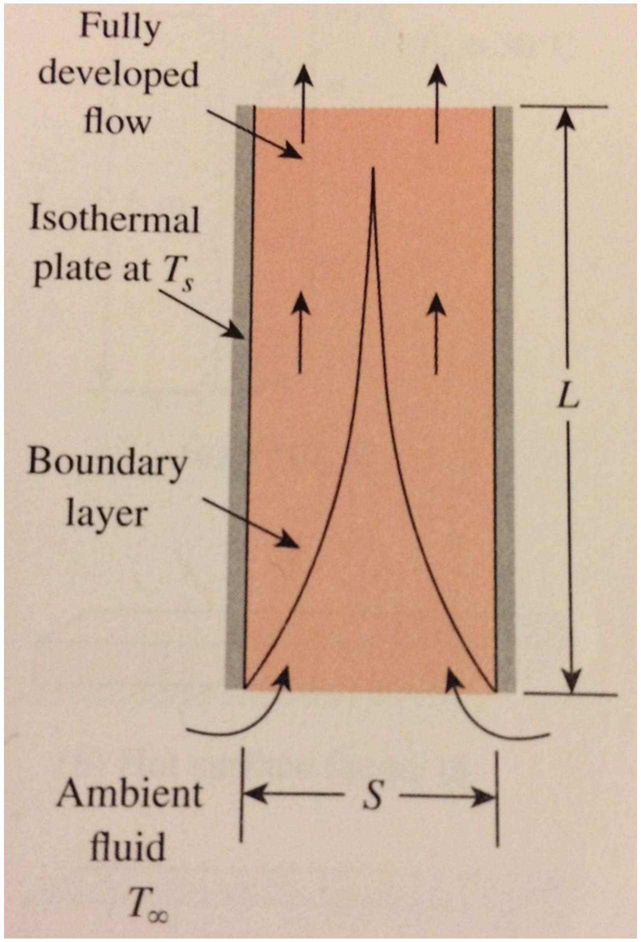
$\nu = 1.749 \times 10^{-5}$ m²/s $\beta = 1/T_f = 1/318$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$

❑ NC in finned surfaces

- **Finned surfaces** of various shapes, called **heat sinks**, are frequently used in the cooling of **electronic devices**.
- **Energy dissipated** by these devices is transferred to the heat sinks by conduction and **from the heat sinks to the ambient air** by **natural or forced convection**.
- **Natural convection** is the **preferred mode of heat transfer** since it involves **no moving parts.**
- The **characteristic length** for NC from vertical finned surfaces of rectangular shape is usually taken to be **the spacing between adjacent fins *S***, although the fin height *L* can also be used.

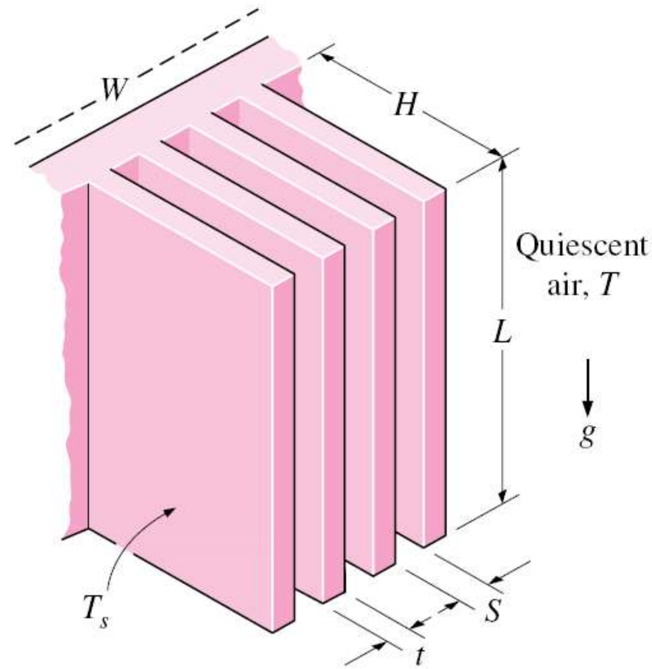


$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \mathrm{Ra}_s\frac{L^3}{S^3} \qquad \mathrm{Ra}_s = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2}\mathrm{Pr}$$

4.3.4 Natural Convection

❑ NC in finned surfaces

$$\dot{Q} = h\,A_s(T_s - T_\infty)$$



- **Closely packed fins**
  - ➢ greater surface area
  - ➢ smaller heat transfer coefficient.

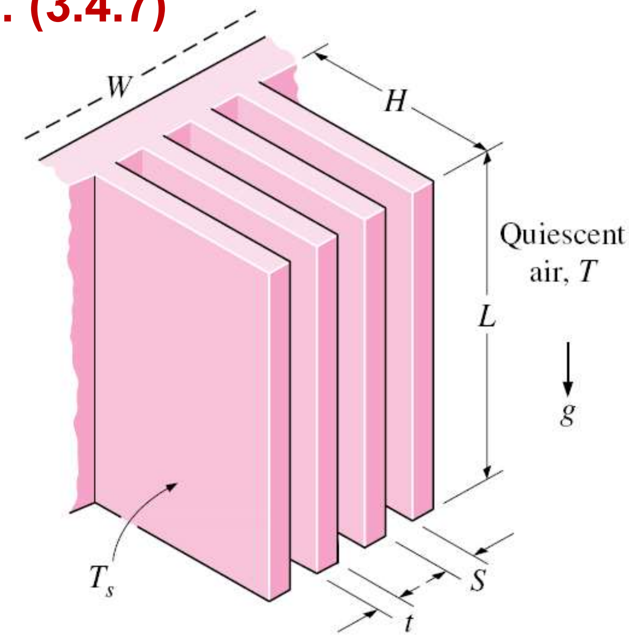
- **Widely spaced fins**
  - ➢ higher heat transfer coefficient
  - ➢ smaller surface area.

- There exists an **optimum fin spacing** for a vertical heat sink in order to maximize heat transfer

❑ NC in finned surfaces

- The recommended relation for the average Nusselt number for vertical **isothermal parallel plates** is:

$$Nu = \frac{hS}{k} = \left[\frac{576}{\left(Ra_s S/L\right)^2} + \frac{2.873}{\left(Ra_s S/L\right)^{0.5}} \right]^{-0.5} \quad \text{… … (3.4.7)}$$



$$\mathrm{Ra}_s = \frac{g\beta (T_s - T_\infty) S^3}{\nu^2} \mathrm{Pr}$$

- Optimum fin spacing for a vertical heat sink

$$S_{opt} = 2.714 \left(\frac{S^3 L}{Ra_s} \right)^{0.25}$$

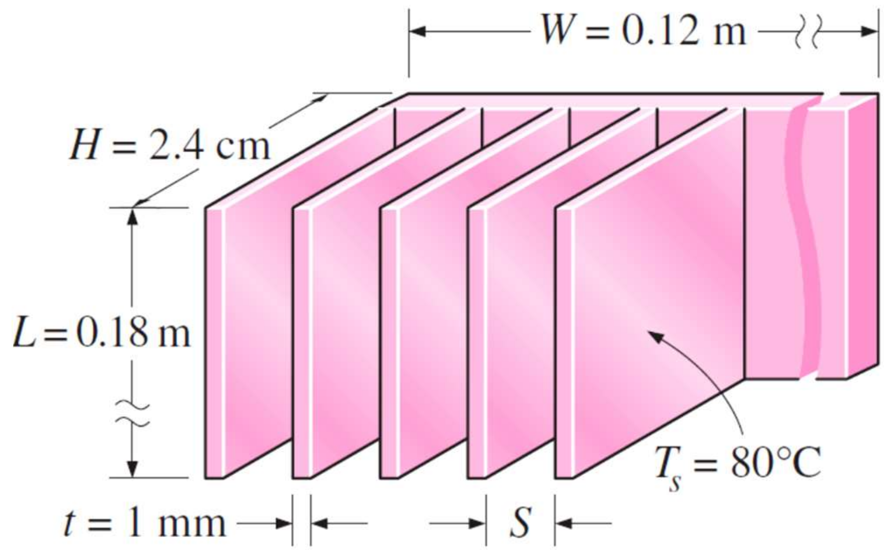
$$\boldsymbol{Nu_{opt} = \frac{hS_{opt}}{k} = 1.307} \quad \text{… … (3.4.8)}$$

EP# 3.12 Cengel et al. Example: 9-2

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.

Assumptions:

1. Steady operating conditions exist.
2. Air is an ideal gas.
3. The atmospheric pressure at that location is 1 atm.
4. The thickness t of the fins is very small relative to the fin spacing S which is considered to be optimum.
5. All fin surfaces are isothermal at base temperature.



EP# 3.12 Soln

Properties

The properties of air at the film temperature of

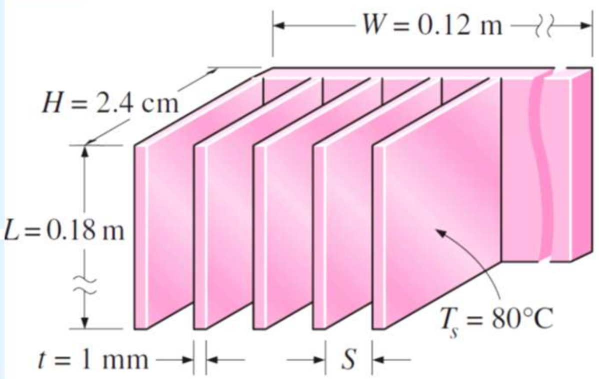
$T_f = (T_s + T_\infty)/2 = (80+30)/2 = 55°C$ and 1 atm pressure are (Table A-15)

$k = 0.02772$ W/m°C Pr = 0.7215

$\nu = 1.846 \times 10^{-5}$ m²/s $\beta = 1/T_f = 1/328$ K⁻¹

Analysis We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)[1/(328\ \text{K})](80 - 30\ \text{K})(0.18\ \text{m})^3}{(1.847 \times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7215)$$
$$= 1.845 \times 10^7$$



$$S_{opt} = 2.714\frac{L}{\text{Ra}_L^{0.25}} = 2.714\frac{0.18\ \text{m}}{(1.845 \times 10^7)^{0.25}} = 7.45 \times 10^{-3}\ \text{m} = 7.45\ \text{mm}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12\ \text{m}}{(0.00745 + 0.001)\ \text{m}} \approx 14\ \text{fins}$$

$$h = \text{Nu}_{opt}\frac{k}{S_{opt}} = 1.307\frac{0.02772\ \text{W/m·K}}{0.00745\ \text{m}} = 4.863\ \text{W/m}^2\text{·K}$$

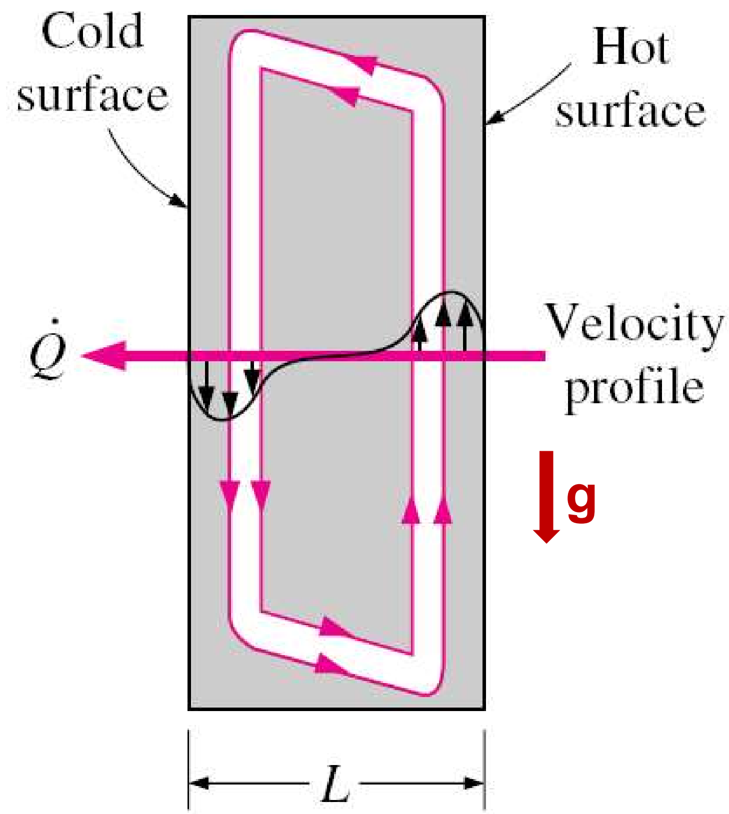
Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty)$$
$$= (4.863\ \text{W/m}^2\text{·K})[2 \times 14(0.18\ \text{m})(0.024\ \text{m})](80 - 30)°\text{C} = 29.4\ \text{W}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 29.4 W.

❑ NC Inside Vertical Enclosures

- A considerable portion of heat loss from a typical residence occurs through the windows.
- We certainly would insulate the windows, if we could, in order to conserve energy.
- *air* is a *better insulator* than most common insulating materials.
- An *enclosure,* which is known as a *double-pane* window is the solution.

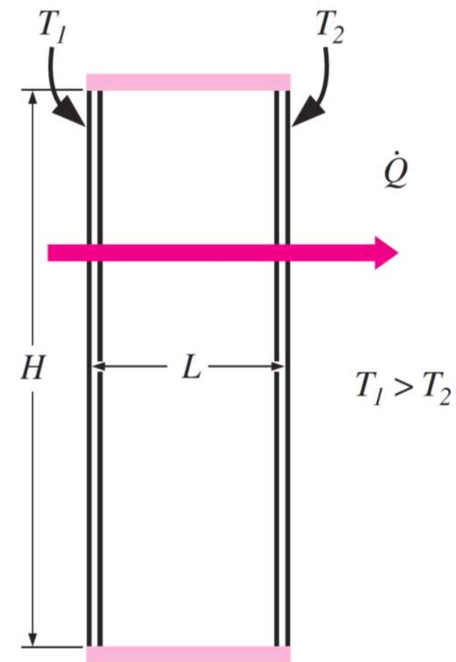


❑ NC Inside Vertical Enclosures

The Rayleigh number is determined from

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr$$

where the characteristic length L_c=L, and T_1 & T_2 are the temperatures of the hot and cold surfaces, respectively.



$$Nu = 0.18\left(\frac{Pr}{0.2 + Pr} Ra_L\right)^{0.29}$$ $1 < H/L < 2$, any Prandtl number, $Ra_L\, Pr/(0.2 + Pr) > 10^3$

$$Nu = 0.22\left(\frac{Pr}{0.2 + Pr} Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$ $2 < H/L < 10$, any Prandtl number, $Ra_L < 10^{10}$

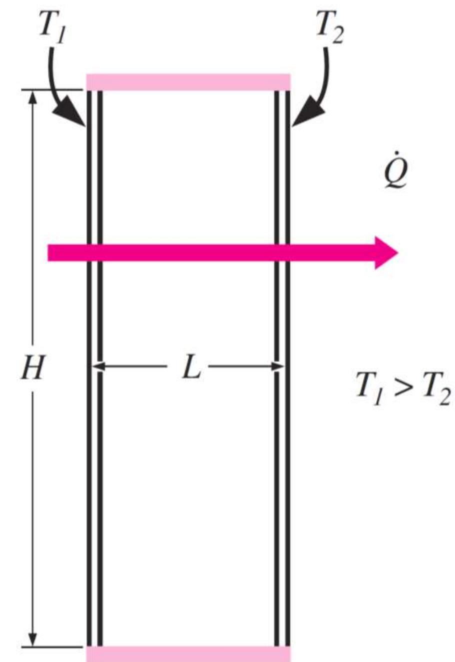
$$Nu = 0.42\, Ra_L^{1/4}\, Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$ $10 < H/L < 40$, $1 < Pr < 2 \times 10^4$, $10^4 < Ra_L < 10^7$

All fluid properties are to be evaluated at the avg fluid temperature T_{avg}=(T_1 + T_2)/2

❑ NC Inside Vertical Enclosures

- When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s \frac{T_1 - T_2}{L_c}$$



- The rate of steady heat conduction across a layer of thickness L_c , area A_s, and thermal conductivity k is expressed as

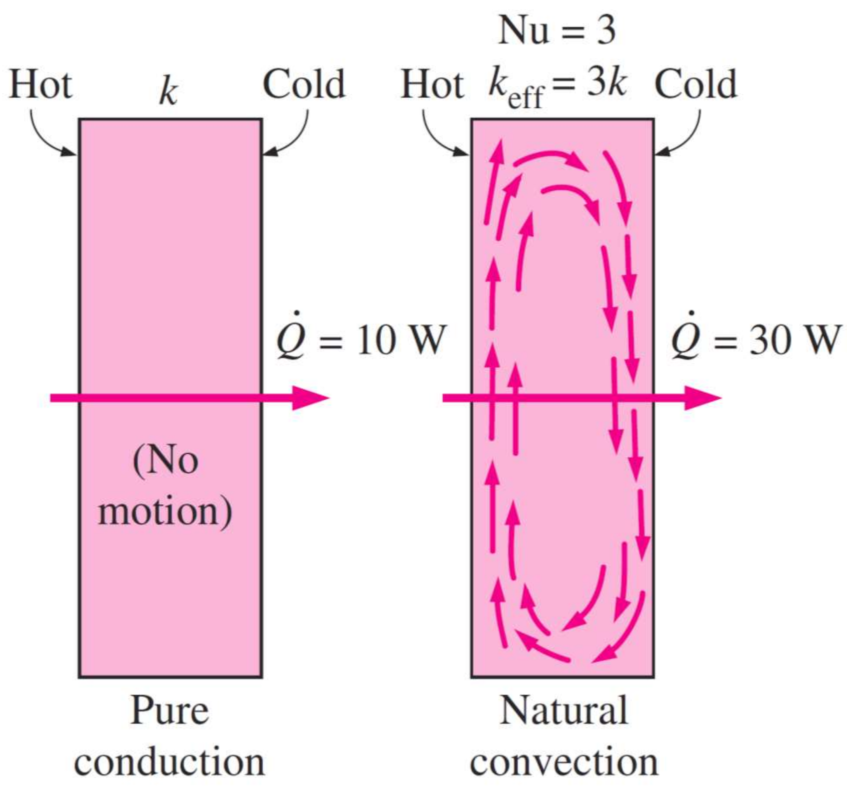
$$\dot{Q}_{\text{cond}} = kA_s \frac{T_1 - T_2}{L_c}$$

- The comparison of convection and conduction relations reveals that the fluid in an enclosure behaves like a fluid whose thermal conductivity is ***k*Nu** as a result of convection currents.

❑ Effective Conductivity

The quantity kNu is called the **effective thermal conductivity** of the enclosure.

$$k_{eff} = k\text{Nu}$$



For the special case of Nu = 1, the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid.