ME265: Thermal Engineering & Heat Transfer

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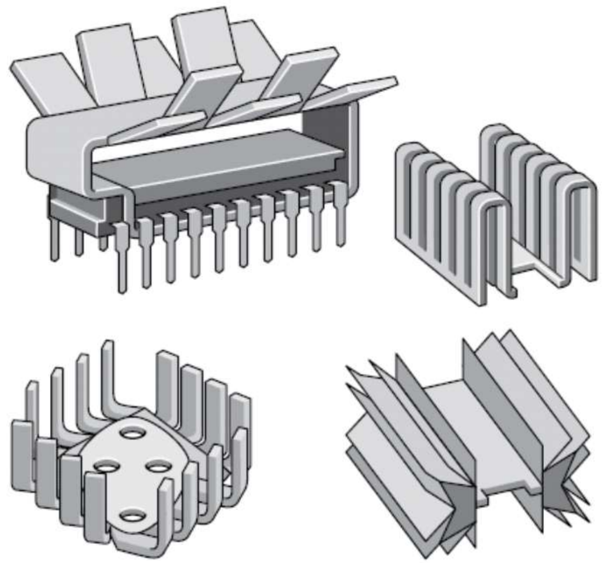
4.2.3.5 Thermal Analysis of fins



Fins are extended surfaces to enhance the rate of heat transfer.

We know from Newton's law cooling

$$\dot{Q} = h\,A_s(T_s - T_\infty)$$

- ✓ Two ways to increase the heat transfer:
  - increasing the heat transfer coefficient *h*
  - increase the surface area A_s

- ✓ *The increase in h* requires the change of surrounding fluid and its condition*, but this approach may not be always practically feasible.*

- ✓ The increase in A_s can easily be done by using of the **extended surfaces** called **fins**

4.2.3.5 Thermal analysis of fins: Fin Equation

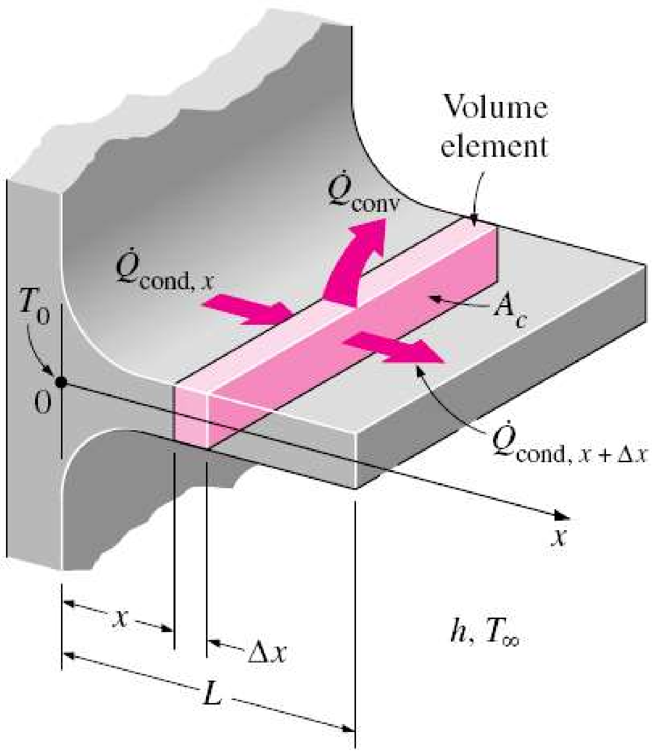


Fig. 2.1: A schematic of fin for Thermal Analysis

Under steady conditions, the energy balance on this volume element can be expressed as follows:

(Rate of heat conduction into the element at *x*) = (Rate of *heat conduction* from the element at *x*+*Δx*) + (Rate of heat convection from the element)

$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv} \quad \text{…… (2.3.1)}$$

By neglecting radiation effect

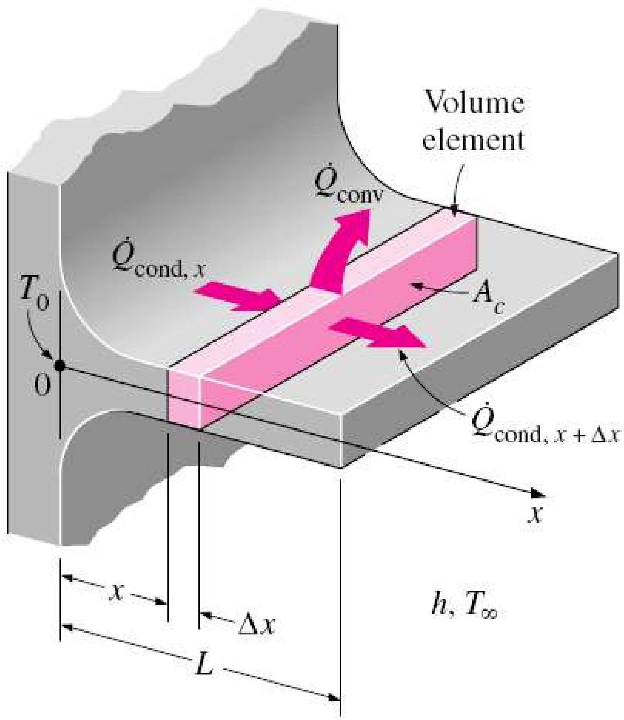


Fig. 2.1: A schematic of fin for Thermal Analysis

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad \text{… … (2.3.4)}$$

Where, $m^2 = \dfrac{hp}{kA_c}$

This is the governing equation of fin. It is a linear, homogeneous, second-order differential equation with constant coefficients.

Therefore, the general solution is as follows:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad \text{… … (2.3.5)}$$

C_1 and C_2 are arbitrary constants and are determined from the BCs.

4.2.3.5 Thermal analysis of fins: Fin BCs

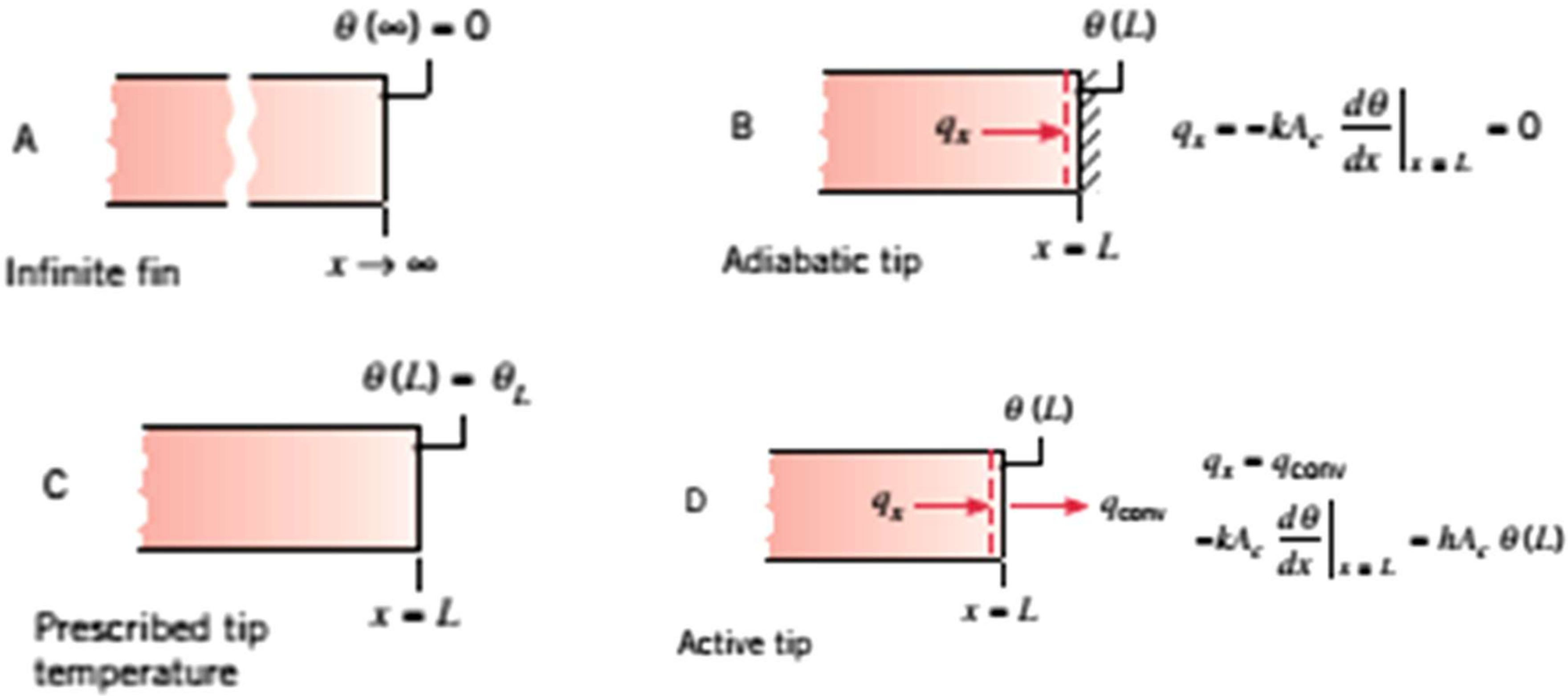


Fig. 2.2: Fin Boundary Conditions

$\theta_b = T_b - T_\propto$; $\theta_L = T_L - T_\propto$

$m = \sqrt{\frac{hp}{kA_c}}$ ……called fin parameter

dimension: m^{-1}

4.2.3.5 Thermal analysis of fins: Fin BCs

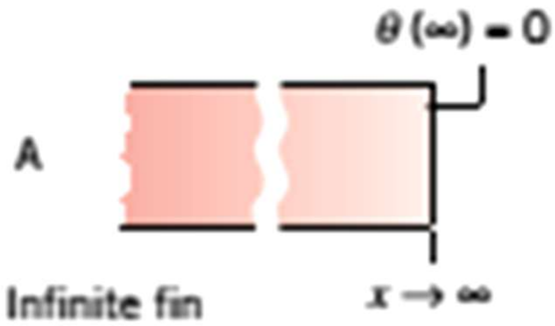


Fig. 2.2(a): Fin with BC—Case A

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \quad \ldots \ldots (2.3.5)$$

C_1 and C_2 are arbitrary constants and are determined from the BCs.

$m = \sqrt{\frac{hp}{kA_c}}$ ……called fin parameter

At x=0,

$$\theta(0) = \theta_b = C_1 + C_2$$

At x=L→∝,

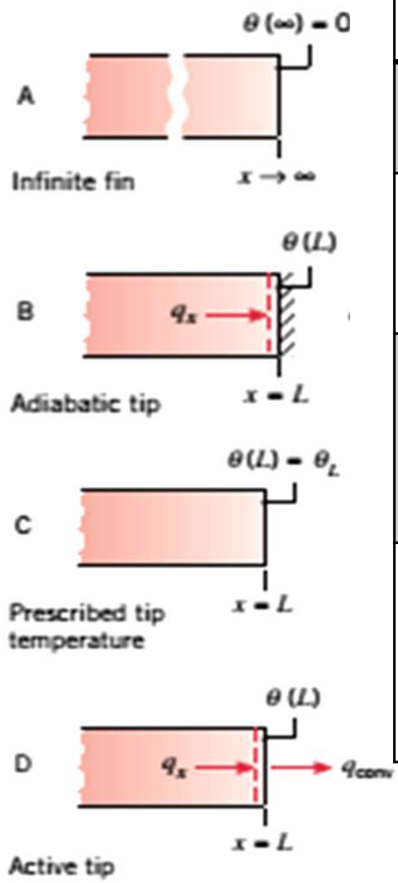
$$\theta(L) = C_1 e^{mL} + C_2 e^{-mL} = 0$$

Therefore, $C_1 = 0$ and $\theta_b = C_2$

The temperature distribution: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \quad \ldots \ldots (2.3.6)$$

The rate of heat transfer: $$\dot{Q} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\left(T_b - T_\infty\right) \quad \ldots \ldots (2.3.7)$$

4.2.3.5 Thermal Analysis of fins: Summary for uniform c/s fin



Case	Temperature Distribution θ/θ_b	Rate of heat transfer $\dot{Q}_f$
A	e^{-mx}	$M = \sqrt{hpkA_c}\,\theta_b$
B	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	$\dfrac{\left(\frac{\theta_L}{\theta_b}\right)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{\cosh mL - \left(\frac{\theta_L}{\theta_b}\right)}{\sinh mL}$
D	$\dfrac{\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mL}\right)\sinh mL}$	$M\dfrac{\sinh mL + \left(\frac{h}{mk}\right)\cosh mL}{\cosh mL + \left(\frac{h}{mL}\right)\sinh mL}$

$\boldsymbol{\theta} = (T - T_\propto);$ $\quad \theta_b = T_b - T_\propto;$ $\quad \theta_L = T_L - T_\propto$

$m = \sqrt{\dfrac{hp}{kA_c}}$ ......called fin parameter

(c) MA Islam, MechE, BUET

4.2.3.5 Analysis of Fins

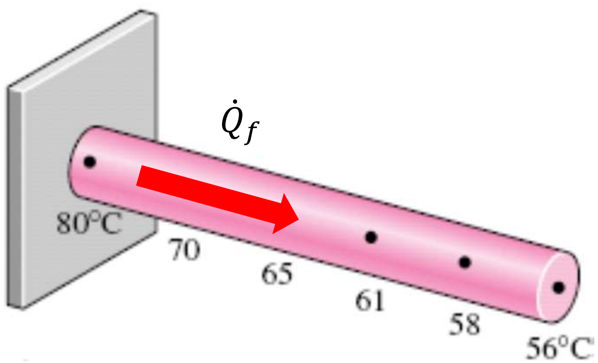
❑ Performance parameters

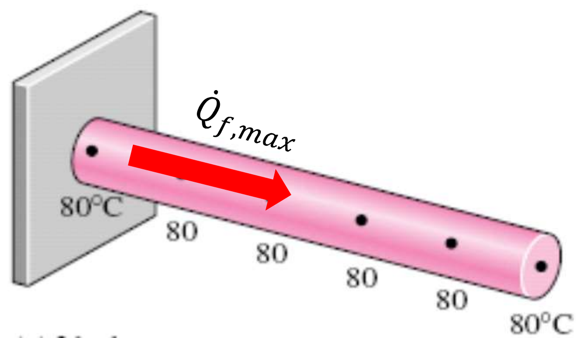
- Fin Efficiency, η_f
- Fin Effectiveness, ε_f

4.2.3.5 Analysis of Fins

❑ Performance parameters: Fin Efficiency, η_f

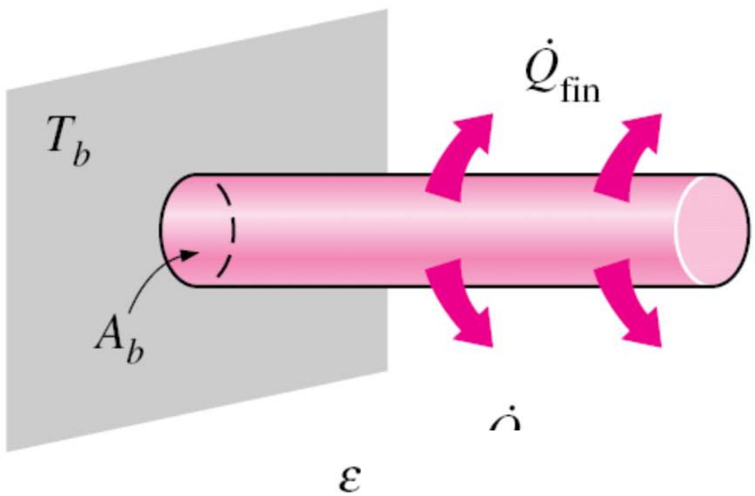
$$\eta_f = \frac{\text{Rate of heat Trasfer with fin}}{\text{Rate of heat transfer if entire fin surface were at base temperature}} = \frac{\dot{Q}_f}{\dot{Q}_{f,max}}$$

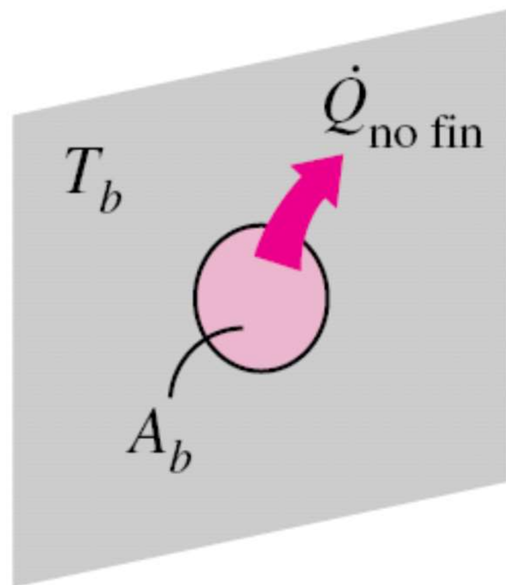




❑ Performance parameters: Fin Effectiveness, ε_f

$$\varepsilon_f = \frac{\text{Rate of heat Trasfer with fin}}{\text{Rate of heat transfer without fin}} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$





Use of fin is not justified unless, $\varepsilon_f \geq 2$

4.2.3.5 Analysis of Fins

❑ Performance parameters

Fin efficiency and Fin effectiveness are related as follows:

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

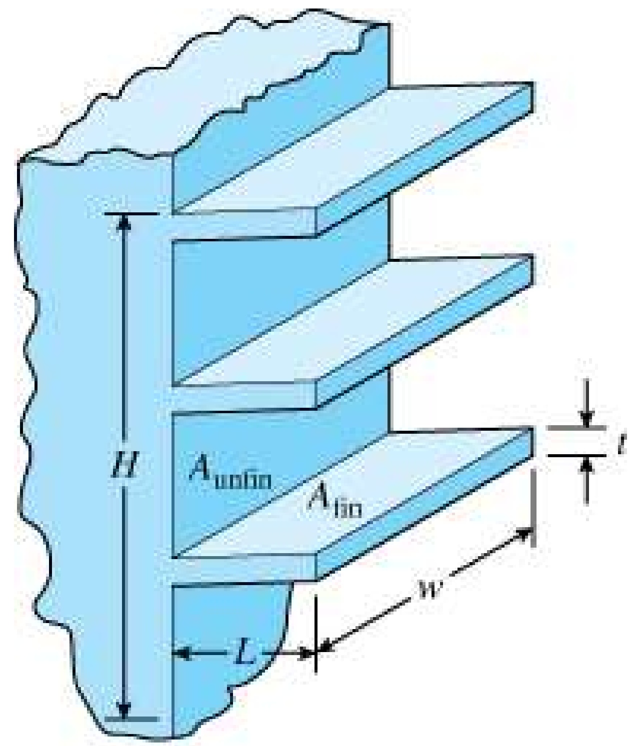
❑ Performance parameters

Overall fin effectiveness:

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$

$$\varepsilon_{fin,overall} = \frac{h(A_{unfin} + \eta_{fin}A_{fin})(T_b - T_\infty)}{hA_{no,fin}(T_b - T_\infty)}$$

$$\varepsilon_{fin,overall} = \frac{A_{unfin} + \eta_{fin}A_{fin}}{A_{no,fin}}$$



$$A_{no\ fin} = w \times H$$
$$A_{unfin} = w \times H - 3 \times (t \times w)$$
$$A_{fin} = 2 \times L \times w + t \times w$$
$$\cong 2 \times L \times w \text{ (one fin)}$$

❑ Design consideration

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

- The ***thermal conductivity k*** of the fin material should be as **high** as possible.
- The ratio $\boldsymbol{p/A_c}$ should be as **high** as possible.
- The use of fins is *most effective* in applications involving a ***low h***

  - ✓ The use of fins is more easily justified when the medium is a ***gas*** instead of a **liquid**

    and

  - ✓ the heat transfer is by ***natural convection*** instead of by **forced convection**.

EP#2.6 Preventing circuit board from overheating

(Cengel et al Example 3-13)

A 15-cm x 20-cm integrated circuit board is to cooled by attaching 4-cm-long aluminum (k=237 W/m.K) fins on one side as shown in the figure. Each fin has a 2-mm x 2-mm square cross-section. The ambient temperature is 25^oC and the convection heat transfer coefficient on fin surface is 20 W/m^2.K. To prevent the circuit board from overheating, the upper surface of the board needs to be at 80^oC or cooler. Determine the number of fins with overall effectiveness of 3.