

# One-Sample Tests of Hypothesis



## Chapter 10



# Learning Objectives

- LO1** Define a hypothesis.
- LO2** Explain the five-step hypothesis-testing procedure.
- LO3** Describe Type I and Type II errors.
- LO4** Define the term test statistic and explain how it is used.
- LO5** Distinguish between a one-tailed and two-tailed hypothesis
- LO6** Conduct a test of hypothesis about a population mean.
- LO7** Compute and interpret a p-value.
- LO8** Conduct a test of hypothesis about a population proportion.
- LO9** Compute the probability of a Type II error.

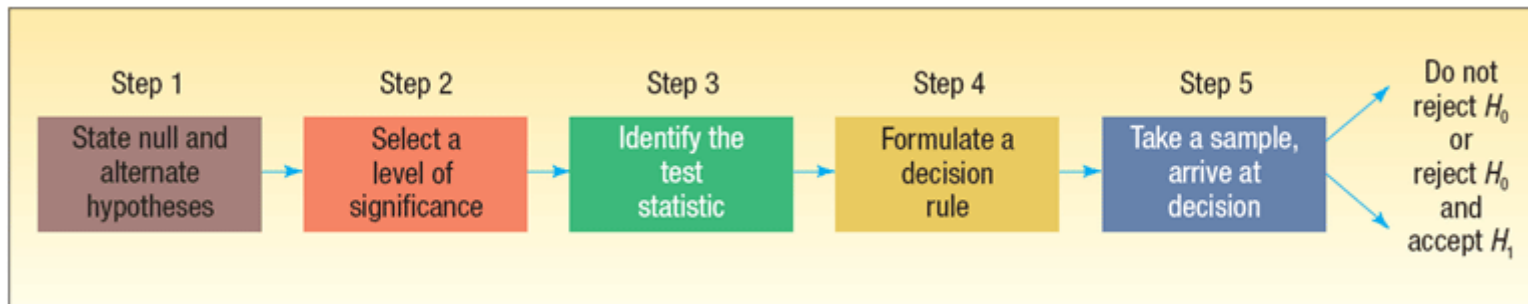
**LO1** Define a hypothesis.

**LO2** Explain the five-step hypothesis-testing procedure.

# Hypothesis and Hypothesis Testing

**HYPOTHESIS** A statement about the value of a population parameter developed for the purpose of testing.

**HYPOTHESIS TESTING** A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.



**TEST STATISTIC** A value, determined from sample information, used to determine whether to reject the null hypothesis.

**CRITICAL VALUE** The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

## LO5 Distinguish between a one-tailed and two-tailed hypothesis

### Important Things to Remember about $H_0$ and $H_1$

- $H_0$ : null hypothesis and  $H_1$ : alternate hypothesis
- $H_0$  and  $H_1$  are mutually exclusive and collectively exhaustive
- $H_0$  is always presumed to be true
- $H_1$  has the burden of proof
- A random sample ( $n$ ) is used to “reject  $H_0$ ”
- If we conclude 'do not reject  $H_0$ ', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not sufficient evidence to reject  $H_0$ ; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of  $H_0$  (e.g. “=”, “≥”, “≤”).
- “≠” “<” and “>” always part of  $H_1$
- In actual practice, the status quo is set up as  $H_0$
- If the claim is “boastful” the claim is set up as  $H_1$  (we apply the Missouri rule – “show me”). Remember,  $H_1$  has the burden of proof
- In problem solving, look for **key words** and convert them into symbols. Some key words include: “**improved, better than, as effective as, different from, has changed**, etc.”

Keywords	Inequality Symbol	Part of:
<i>Larger (or more) than</i>	>	$H_1$
<i>Smaller (or less)</i>	<	$H_1$
<i>No more than</i>	≤	$H_0$
<i>At least</i>	≥	$H_0$
<i>Has increased</i>	>	$H_1$
<i>Is there difference?</i>	≠	$H_1$
<i>Has not changed</i>	=	$H_0$
<i>Has “improved”, “is better than”. “is more effective”</i>	See left text	$H_1$

**LO6** Conduct a test of hypothesis about a population mean.

# Hypothesis Setups for Testing a Mean ( $\mu$ ) or a Proportion ( $\pi$ )

MEAN

$H_0: \mu = \text{value}$   
 $H_1: \mu \neq \text{value}$

Reject  $H_0$  if:  
 $|Z| > Z_{\alpha/2}$   
 $|t| > t_{\alpha/2, n-1}$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$H_0: \mu \geq \text{value}$   
 $H_1: \mu < \text{value}$

Reject  $H_0$  if:  
 $Z < -Z_{\alpha}$   
 $t < -t_{\alpha, n-1}$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$H_0: \mu \leq \text{value}$   
 $H_1: \mu > \text{value}$

Reject  $H_0$  if:  
 $Z > Z_{\alpha}$   
 $t > t_{\alpha, n-1}$

PROPORTION

$H_0: \pi = \text{value}$   
 $H_1: \pi \neq \text{value}$

Reject  $H_0$  if:  
 $|Z| > Z_{\alpha/2}$

$H_0: \pi \geq \text{value}$   
 $H_1: \pi < \text{value}$

Reject  $H_0$  if:  
 $Z < -Z_{\alpha}$

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$

$H_0: \pi \leq \text{value}$   
 $H_1: \pi > \text{value}$

Reject  $H_0$  if:  
 $Z > Z_{\alpha}$

# Testing for a Population Mean with a Known Population Standard Deviation- Example

## EXAMPLE

Jamestown Steel Company manufactures and assembles desks and other office equipment. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of **200** and a standard deviation of **16**. Recently, new production methods have been introduced and new employees hired. The VP of manufacturing would like to investigate whether there has been a **change** in the weekly production of the Model A325 desk.

**Step 1: State the null hypothesis and the alternate hypothesis.**

$$H_0: \mu = 200$$

$$H_1: \mu \neq 200$$

(note: keyword in the problem “has changed”)

**Step 2: Select the level of significance.**

$$\alpha = 0.01 \text{ as stated in the problem}$$

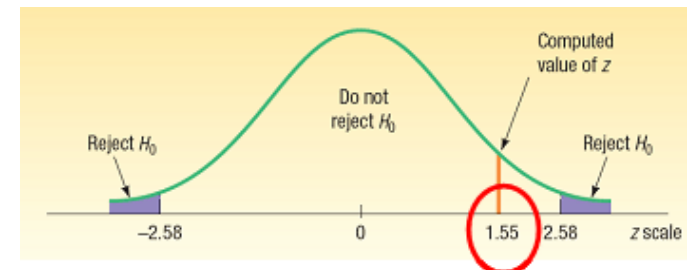
**Step 3: Select the test statistic.**

Use Z-distribution since  $\sigma$  is known

**Step 4: Formulate the decision rule.**

Reject  $H_0$  if  $|Z| > Z_{\alpha/2}$

$$\begin{aligned} |Z| &> Z_{\alpha/2} \\ \left| \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \right| &> Z_{\alpha/2} \\ \left| \frac{203.5 - 200}{16 / \sqrt{50}} \right| &> Z_{.01/2} \\ 1.55 &\text{ is not } > 2.58 \end{aligned}$$



**Step 5: Make a decision and interpret the result.**

Because 1.55 does not fall in the rejection region,  $H_0$  is not rejected. We conclude that the population mean is not different from 200. So we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the plant has changed from 200 per week.

# Testing for a Population Mean with a Known Population Standard Deviation- Another Example

Suppose in the previous problem the vice president wants to know whether there has been an **increase** in the number of units assembled. To put it another way, can we conclude, because of the improved production methods, that the mean number of desks assembled in the last 50 weeks was **more than 200**?

Recall:  $\sigma=16$ ,  $n=200$ ,  $\alpha=.01$

**Step 1: State the null hypothesis and the alternate hypothesis.**

$$H_0: \mu \leq 200$$

$$H_1: \mu > 200$$

(note: keyword in the problem “an **increase**”)

**Step 2: Select the level of significance.**

$\alpha = 0.01$  as stated in the problem

**Step 3: Select the test statistic.**

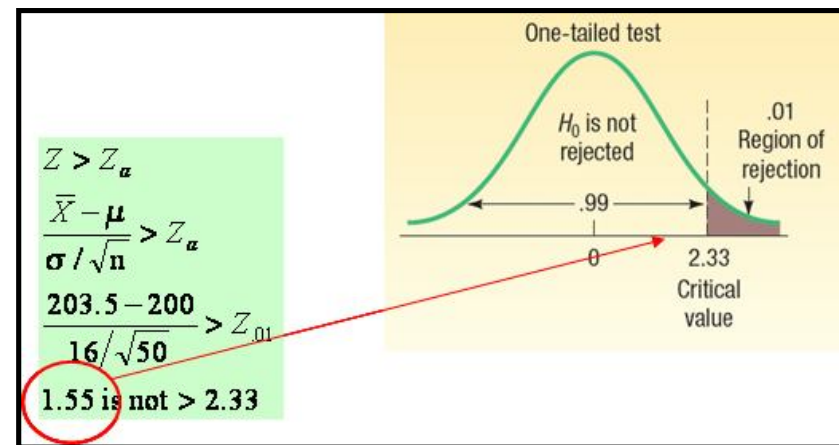
Use Z-distribution since  $\sigma$  is known

**Step 4: Formulate the decision rule.**

Reject  $H_0$  if  $Z > Z_\alpha$

**Step 5: Make a decision and interpret the result.**

Because 1.55 does not fall in the rejection region,  $H_0$  is not rejected. We conclude that the average number of desks assembled in the last 50 weeks is not more than 200



# Testing for the Population Mean: Population Standard Deviation Unknown

When the population standard deviation ( $\sigma$ ) is unknown, the sample standard deviation ( $s$ ) is used in its place the  $t$ -distribution is used as test statistic, which is computed using the formula:

TESTING A MEAN,  $\sigma$  UNKNOWN

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

with  $n - 1$  degrees of freedom, where:

$\bar{X}$  is the sample mean.

$\mu$  is the hypothesized population mean.

$s$  is the sample standard deviation.

$n$  is the number of observations in the sample.

## EXAMPLE

The McFarland Insurance Company Claims Department reports the **mean** cost to process a claim is **\$60**. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of **26** claims processed last month. The sample information is reported below.

At the **.01** significance level is it reasonable a claim is **now less than \$60**?

\$45	\$49	\$62	\$40	\$43	\$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				

# Testing for the Population Mean: Population Standard Deviation Unknown - Example

**Step 1: State the null hypothesis and the alternate hypothesis.**

$$H_0: \mu \geq \$60$$

$$H_1: \mu < \$60$$

**Step 2: Select the level of significance.**

$\alpha = 0.01$  as stated in the problem

**Step 3: Select the test statistic.**

Use t-distribution since  $\sigma$  is unknown

**Step 4: Formulate the decision rule.**

$$\text{Reject } H_0 \text{ if } t < -t_{\alpha, n-1}$$

**Step 5: Make a decision and interpret the result.**

Because -1.818 does not fall in the rejection region,  $H_0$  is not rejected at the .01 significance level. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60. The difference of \$3.58 (\$56.42 - \$60) between the sample mean and the population mean could be due to sampling error.

TABLE 10-1 A Portion of the t Distribution Table

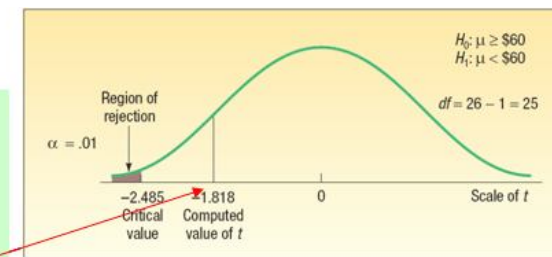
Confidence Intervals						
	80%	90%	95%	98%	99%	99.9%
df	Level of Significance for One-Tailed Test, $\alpha$					
	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of Significance for Two-Tailed Test, $\alpha$					
	0.20	0.10	0.05	0.02	0.01	0.001
∴	∴	∴	∴	∴	∴	∴
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646

$$t < -t_{\alpha, n-1}$$

$$\frac{\bar{X} - \mu}{s / \sqrt{n}} < -t_{\alpha, n-1}$$

$$\frac{\$56.42 - \$60}{\$10.04 / \sqrt{26}} < -t_{.01, 26-1}$$

$$-1.818 \text{ is not } < -2.485$$



## Type of Errors and $p$ -value in Hypothesis Testing

- **Type I Error -**
  - Defined as the probability of rejecting the null hypothesis when it is actually true.
  - This is denoted by the Greek letter " $\alpha$ "
  - Also known as the *significance level* of a test
  
- **Type II Error:**
  - Defined as the probability of "accepting" the null hypothesis when it is actually false.
  - This is denoted by the Greek letter " $\beta$ "
  
- **$p$ -VALUE** is the probability of observing a sample value as extreme as, or more extreme than, the value observed, given that the null hypothesis is true.
  
- In testing a hypothesis, we can also compare the  $p$ -value to the significance level ( $\alpha$ ).
  
- Decision rule using the  $p$ -value:  
**Reject  $H_0$  if  $p$ -value < significance level**

### EXAMPLE $p$ -Value

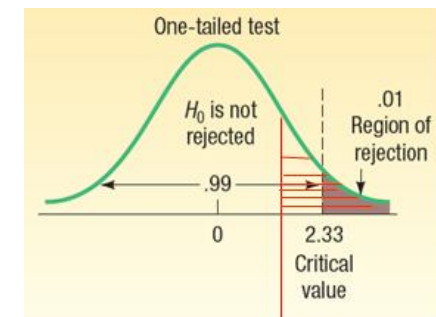
Recall the last problem where the hypothesis and decision rules were set up as:

$$H_0: \mu \leq 200$$

$$H_1: \mu > 200$$

Reject  $H_0$  if  $Z > Z_\alpha$   
 where  $Z = 1.55$  and  $Z_\alpha = 2.33$

Reject  $H_0$  if  $p$ -value <  $\alpha$   
 $0.0606$  is not <  $0.01$



1.55

Conclude: Fail to reject  $H_0$

$$P(Z > 1.55) = .5000 - .4394$$

$$P\text{-value} = .0606$$

## LO8 Conduct a test of hypothesis about a population proportion.

# Tests Concerning Proportion using the z-Distribution

- A **Proportion** is the fraction or percentage that indicates the part of the population or sample having a particular trait of interest.
- The sample proportion is denoted by  $p$  and is found by  $x/n$
- It is assumed that the binomial assumptions discussed in Chapter 6 are met:
  - (1) the sample data collected are the result of counts;
  - (2) the outcome of an experiment is classified into one of two mutually exclusive categories—a “success” or a “failure”;
  - (3) the probability of a success is the same for each trial; and (4) the trials are independent
- Both  $n\pi$  and  $n(1-\pi)$  are at least 5.
- When the above conditions are met, the normal distribution can be used as an approximation to the binomial distribution
- The test statistic is computed as follows:

TEST OF HYPOTHESIS, ONE PROPORTION

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \quad [10-3]$$

where:

- $\pi$  is the population proportion.
- $p$  is the sample proportion.
- $n$  is the sample size.

# Test Statistic for Testing a Single Population Proportion - Example

## EXAMPLE

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office and plans to conduct a survey of 2,000 registered voters in the northern section of the state. Using the hypothesis-testing procedure, assess the governor's chances of reelection.

**Step 1: State the null hypothesis and the alternate hypothesis.**

$$H_0: \pi \geq .80$$

$$H_1: \pi < .80$$

(note: keyword in the problem “at least”)

**Step 2: Select the level of significance.**

$\alpha = 0.01$  as stated in the problem

**Step 3: Select the test statistic.**

Use Z-distribution since the assumptions are met and  $n\pi$  and  $n(1-\pi) \geq 5$

**Step 4: Formulate the decision rule.**

Reject  $H_0$  if  $Z < -Z_\alpha$

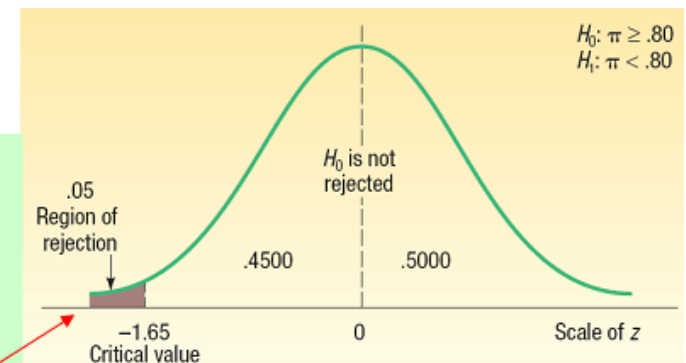
$$Z < -Z_\alpha$$

$$\frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} < -Z_\alpha$$

$$\frac{1,550}{2,000} - .80 < -1.65$$

$$\frac{.80(1-.80)}{\sqrt{2,000}}$$

$$-2.80 < -1.65$$



**Step 5: Make a decision and interpret the result.**

The computed value of  $z$  (2.80) is in the rejection region, so the null hypothesis is rejected at the .05 level. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

## LO9 Compute the probability of a Type II error.

# Type II Error

- Recall **Type I Error**, the level of significance, denoted by the Greek letter “ $\alpha$ ”, is defined as the probability of rejecting the null hypothesis when it is actually true.
- Type II Error**, denoted by the Greek letter “ $\beta$ ”, is defined as the probability of “accepting” the null hypothesis when it is actually false.

### EXAMPLE

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is 10,000 psi and that the standard deviation,  $\sigma$ , is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: “Take a sample of 100 steel bars. At the .05 significance level if the sample mean strength falls between 9,922 psi and 10,078 psi, accept the lot. Otherwise the lot is to be rejected.”

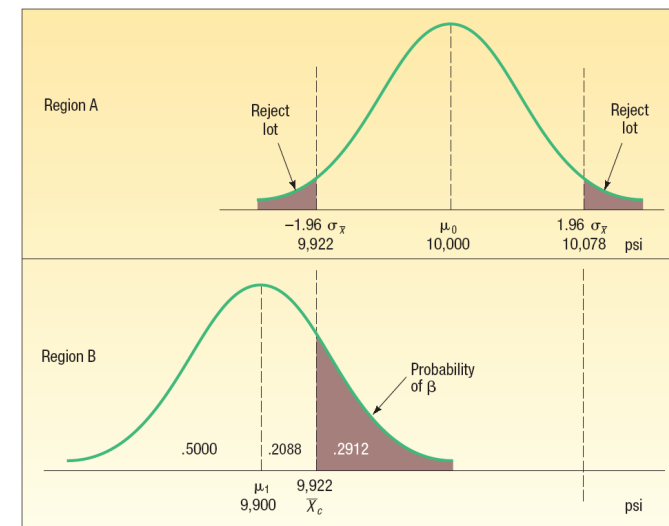
The number of standard units (z value) between the mean of the incoming lot (9,900), designated by  $\mu_1$ , and  $\bar{X}_c$ , representing the critical value for 9,922, is computed by:

$$\text{TYPE II ERROR} \quad z = \frac{\bar{X}_c - \mu_1}{\sigma/\sqrt{n}} \quad [10-4]$$

With  $n = 100$  and  $\sigma = 400$ , the value of z is 0.55:

$$z = \frac{\bar{X}_c - \mu_1}{\sigma/\sqrt{n}} = \frac{9,922 - 9,900}{400/\sqrt{100}} = \frac{22}{40} = 0.55$$

The area under the curve between 9,900 and 9,922 (a z value of 0.55) is .2088. The area under the curve beyond 9,922 pounds is .5000 – .2088, or .2912; this is the probability of making a Type II error—that is, accepting an incoming lot of steel bars when the population mean is 9,900 psi.



# Type I and Type II Errors Illustrated

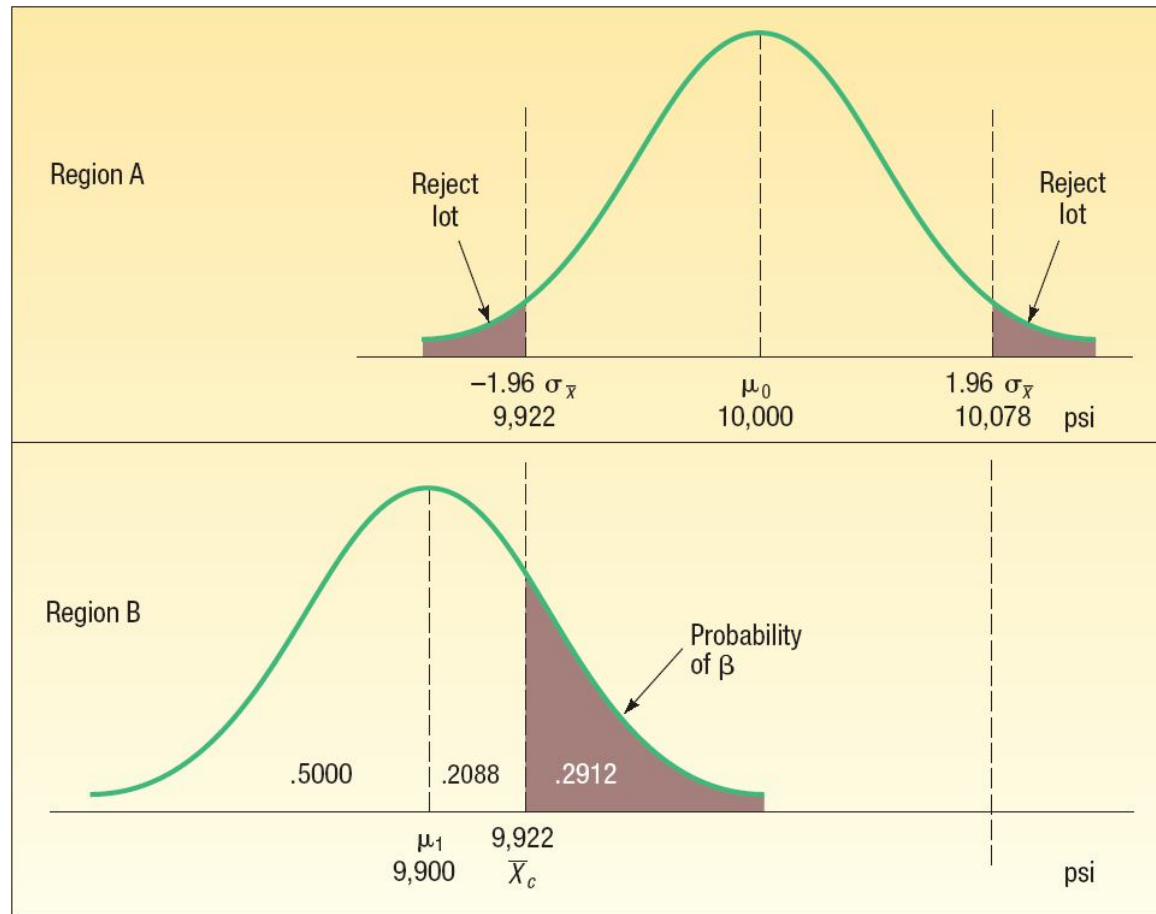


CHART 10-9 Charts Showing Type I and Type II Errors

Dr. Sultana Parveen, Professor, IPE Dept, BUET

# Type II Errors For Varying Mean Levels

**TABLE 10-4** Probabilities of a Type II Error for  $\mu_0 = 10,000$  Pounds and Selected Alternative Means, .05 Level of Significance

Selected Alternative Mean (pounds)	Probability of Type II Error ( $\beta$ )	Probability of Not Making a Type II Error ( $1 - \beta$ )
9,820	.0054	.9946
9,880	.1469	.8531
9,900	.2912	.7088
9,940	.6736	.3264
9,980	.9265	.0735
10,000	— *	—
10,020	.9265	.0735
10,060	.6736	.3264
10,100	.2912	.7088
10,120	.1469	.8531
10,180	.0054	.9946

\*It is not possible to make a Type II error when  $\mu = \mu_0$ .