6.

(a)

January 2021

$D = 3000$

$\sigma_D = 500$

$L = 2$

$T = 4$

$CSL = 95$

Now,

Mean Demand During T+L period $= D(T+L)$

$= 3000 \times 6$

$= 18,000$

Standard deviation $\sigma_{T+L} = \sqrt{T+L}\ \sigma_D$

$= \sqrt{6} \times 500$

$= 1224.745$

Safety stock $= F_S^{-1}(CSL) \times \sigma_{T+L}$

$= F_S^{-1}(0.95) \times 1224.745$

$= 1.64485 \times 1224.745$

$= 2015$ unit

$OUL = D_{T+L} + SS$

$= 18000 + 2015$

$= 20,015$ unit

(b)

Let, D_i = Mean weekly Demand $\quad i = 1, 2, 3 - - .$

σ_i = Standard deviation

The aggregate demand

$$D^C = \sum_{i}^{K} D_i = KD$$

$$\text{Standard deviation} = \sqrt{\sum_{i=1}^{K} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j}$$

If σ equal and $\rho_{ij} = 0$

$$\sigma_D^C = \sqrt{K}\,\sigma_D$$

without aggregation Demand $= KD$

Standard deviation $= K\sigma$

clearly $\sigma_D^C < \sigma_D$

$$\text{Safety inventory} = \sum_{i=1}^{K} F^{-1}(CSL) \times \sqrt{L}\,\sigma_D$$

$$\text{Safety inventory} = F^{-1}(CSL) \times \sqrt{L} \times \sqrt{K}\,\sigma_D$$

7.

(a)

$q_0 = 0$, $q_1 = 10{,}000$, $q_2 = 20{,}000$

$c_0 = 200$, $c_1 = 195$, $c_2 = 190$

$v_0 = 0$, $v_1 = c_0(q_1 - q_0) = 200 \times 10{,}000$

$= 2000{,}000$

$v_2 = 2000{,}000 + c_1(q_2 - q_1) = 3950{,}000$

$$Q_0 = \sqrt{\frac{2D(S + v_0 - q_0 c_0)}{h c_0}}$$

$$= \sqrt{\frac{2 \times 5000 \times 12\,(500 + 0 - 0)}{0.25 \times 200}} = 1096 \text{ unit}$$

$$Q_1 = \sqrt{\frac{2 \times 5000 \times 12\,(500 + 2000{,}000 - 10000 \times 195)}{0.25 \times 195}}$$

$= 11150$ unit

$$Q_2 = \sqrt{\frac{2 \times 5000 \times 12 \times (500 + 3950{,}000 - 20{,}000 \times 190)}{0.25 \times 190}}$$

$= 19499$ unit

Setting $Q_0 = 1096$,

$Q_1 = 11150$

$Q_2 = 20{,}000$

$$TC_0 = \frac{12 \times 5000}{1096} \times 500 + \frac{12 \times 5000}{1096}\left[0 + (1096-0) \times 200\right] + \frac{(0 + (1096-0) \times 200) \times 0.25}{2}$$

$$= \$120\,54772$$

$$TC_1 = \frac{12 \times 5000 \times 500}{11150} + \frac{12 \times 5000}{11150}\left[2000{,}000 + (11150 - 10{,}000)\,195\right]$$

$$+ \frac{\left[2000{,}000 + (11150 - 10{,}000) \times 192\right] \times 0.25}{2}$$

$$= 12249780.$$

$$TC_2 = \frac{12 \times 5000 \times 500}{20{,}000} + \frac{12 \times 5000}{20{,}000}\left[3950{,}000 + (20{,}000 - 20000)\,190\right]$$

$$+ \frac{\left[3950000 + (20{,}000 - 20{,}000) \times 190\right] 0.25}{2}$$

$$= \boxed{12345250}$$

$\mu = 100, \sigma = 40)$

7.

(b)

$C = 250, \quad P = 300, \quad S = 50 - 20 = 30$

$C_u = P - C = 300 - 250 = 50$

$C_o = C - S = 250 - 30 = 220$

$$CSL = \frac{C_u}{C_o + C_u} = \frac{50}{50 + \cancel{270}} = \cancel{0.15625}\ 0.18518$$

$$O^* = F^{-1}(CSL, \mu, \sigma)$$

$$= F^{-1}(\cancel{0.15625}\ 0.18518, 100, 40)$$

$$= \cancel{59.6}\ 65 \approx 65$$

Expected profit

$$= (P-S)\mu \text{ NORMDIST}\left(\frac{O-\mu}{\sigma}, 0, 1, 1\right) + (P-S)\sigma \text{ NORMDIST}\left(\frac{O-\mu}{\sigma}, 0, 1, 0\right)$$

$$- O(C-S) \text{ NORMDIST}(O, \mu, \sigma, 1) + O(P-C)[1 - \text{NORM.}]$$

$$= (300-30) \times 100 \times 0.19 + (300-30) \times 40 \times 0.272$$

$$- 65 \times 220 \times 0.19 + 65 \times 50 \times (1 - 0.19)$$

$$= \$7983.1$$

8.

(a)

$\sigma_D = 300$. $D = 300$

$SS = F^{-1}(CSL)\sqrt{L} \times \sigma_D$

$= F^{-1}(0.95) \times \sqrt{4} \times 300$

$= 1.64485 \times 2 \times 300$

$= 987$ unit

$= 987 \times 25 = 24675$

$D^c = 300 \times 25 = 7500$

$\sigma_D^c = \sqrt{25} \times 300 = 1500$

$SS = 1.64485 \times 2 \times 1500 = 4935$ unit

Saving in safety inventory $= 24675 - 4935$

$= 19740$ unit

$\sigma_D^c = \sqrt{1500 + 300 \times \sqrt{25 \times 24}} = \cancel{6696.152}$ $\cancel{8848.469}$ 5408

$SS = \cancel{29108.8}$ ~~unit~~

$= \cancel{22029}$ unit

$= 17792$ unit

2018-19

6.

(b)

Demand	Probability	Cumulative Probability Dem. D_i	Probability $F \geq D_i$
			1
100	0.1	0.1	0.9
200	0.15	0.25	0.75
300	0.15	0.40	0.6
400	0.2	0.6	0.4
500	0.15	0.75	0.25
600	0.15	0.9	0.1
700	0.1	1.	0

$C_o = 10-5 = 5$

$C_u = 30-10 = 20$

$$CSL = \frac{20}{20+5} = 0.8$$

$$O = F^{-1}(CSL, \mu, \sigma)$$

Demand Benifit Cost Cont.

100

Expected loss = $\sum D_i P_i$
$= 400$

7.

(a) Annual Holding and ordering

$= \frac{Q}{2} hC + \frac{D}{Q} S$

$= \frac{390}{2} \times 0.25 \times 60 + \frac{18 \times 52}{390} \times 45$

$= 3033$

$= \frac{468}{2} \times 0.25 \times 60 + \frac{18 \times 52}{468} \times 45$

$= 3600$

$SS = F_s^{-1}(CSL) \times \sqrt{L}\, \sigma_D$

$= 1.28 \times \sqrt{2} \times 5 = 9.05$

$ROP = SS + D \times L$

$= 9 + 18 \times 2$

$= 45$ unit

(b) Book example

2017-18

3. (b)

$$n = \sqrt{\frac{\Sigma D h C}{2S}}$$

$$= \sqrt{\frac{1500 \times 0.15 \times 5 + 1000 \times 10 \times 0.15 + 500 \times 15 \times 0.15}{2 \times (60 + 30 + 10)}}$$

$= 4.33$ order

Common ordering cost per order $= \frac{612.4}{4.33}$

$= 141.43$

4.

(b)

$D = 2000.$ $\sigma_D = 200.$

$S_L = 1$, $L = 3$ $CSL = 0.975$

$$\sigma_D^c = \sqrt{L\sigma_D^2 + D^2 S_L^2}$$

$$= \sqrt{3 \times 200^2 + 2000^2 \times 1^2} = 2029.778$$

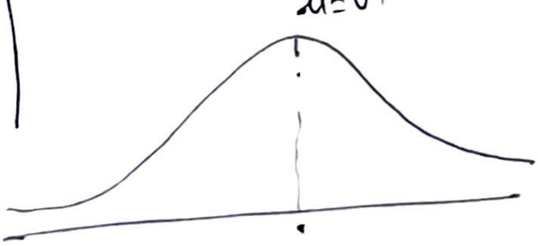
$SS = F^{-1}(0.975) \times 2029.778$

$= 3989$ unit

Days $= \frac{3989}{2000}$

≈ 1.98 days

$\mu = 0$



2016-17

1.

(b)

Demand during lead time = $\frac{8000}{0.5}$ = 16,000

Demand per day = $\frac{730,000}{365}$ = 2000

Lead time = $\frac{16,000}{2000}$ = 8 day.

In transit time = (8-2) = 6 day.

Annual Holding Cost = $\left(\frac{36,500}{2} + 8000\right) \times 0.15 \times 100$

= \$ 393750

Annual Ordering Cost = $\frac{730,000}{36500} \times 500$

= \$ 10,000

6.

(a) $Q = \sqrt{\frac{2DS}{hc}}$

$= \sqrt{\frac{2 \times 3100 \times (110+25)}{0.11 \times 34.99}} = 467$ unit

$Q_0 = 467$ unit

$Q_1 = \sqrt{\frac{2 \times 3100 \times (110+25)}{0.11 \times (34.99 - 0.03 \times 34.99)}} = 474$

$Q_2 = \sqrt{\frac{2 \times 3100 \times 135}{0.11 \times (34.99 - 0.05 \times 34.99)}} = 479$

$Q_0 = 467$

$Q_1 = 500$

$Q_2 = 2000$

$TC_0 = 3100 \times 34.99 + \frac{467}{2} \times 0.11 \times 34.99 + \frac{3100}{467} \times 135$

$TC_1 = 3100 \times (34.99 - 0.03 \times 34.99) + \frac{500}{2} \times 0.11$

6.

(a)

$$Q = \sqrt{\frac{2DS}{hc}}$$

$$= \sqrt{\frac{2 \times 3100 \times (110+25)}{0.11 \times 34.99}} = 467 \text{ unit}$$

$Q_0 = 467$ unit

$$Q_1 = \sqrt{\frac{2 \times 3100 \times (110+25)}{0.11 \times (34.99 - 0.03 \times 34.99)}} = 474$$

$$Q_2 = \sqrt{\frac{2 \times 3100 \times 135}{0.11 \times (34.99 - 0.05 \times 34.99)}} = 479$$

$Q_0 = 467$

$Q_1 = 500$

$Q_2 = 2000$

$$TC_0 = 3100 \times 34.99 + \frac{467}{2} \times 0.11 \times 34.99 + \frac{3100}{467} \times 135$$

$$TC_1 = 3100 \times (34.99 - 0.03 \times 34.99) + \frac{500}{2} \times 0.11$$

7.

(a) $D = 5000 \quad \sigma_D = 4000 \quad , \quad CSL = 0.99$

$SS = F^{-1}(CSL) \times \sqrt{36} \times 4000$

$= 2.326 \times 6 \times 4000 = 55824$

$Days = \frac{55824}{5000} = 11.16$

$SS = 2.326 \times 2 \times 4000 = 18608$

$day = 3.72$

$cycle \ Inv = \frac{100,000}{2}$ day = 10

$cycle \ Inv = 5000$ day = 1.05

$36 \times 5000 = 180,000$

$Total \ Cost = 55824 \times 0.2 \times 100 + 5000 \times 0.2 \times 100 + 0.5 \times 365 \times 500$

$= 3029147$

$Total = 18608 \times 0.2 \times 100 + 2500 \times 0.2 \times 100 + 0.5 \times 365 \times 500$

$p = 35, \quad c = 20. \quad s = 15 - 2 = 13$

$C_u = p - c = 35 - 20 = 15$

$C_o = c - s = 20 - 13 = 7$

$$CSL = \frac{C_u}{C_o + C_u} = \frac{15}{22} = 0.6818.$$

Mean Demand $= 30{,}000 + 30{,}000 = 60{,}000$

Standard deviation $= \sqrt{3000^2 + 8 \times 4000^2} = 8544$

$\cancel{O = F^{-1}(CSL) \times \sigma_L}$

$O = F^{-1}(CSL, \mu, \sigma)$

$= F^{-1}(0.6818, 60{,}000, 8544)$

$= 32364, \quad 11890$

$$\sigma_L = \sqrt{L\sigma_D^2 + D^2 S_L^2}$$

$$6037.39^2 = L \times 300^2 + 3000^2 \times 4$$

2015-16

$$L = 5$$

3.

(b) $ROP = D \times L + SS$

$= 3000 \times 5 + 9931.49$

$= 24931.49$ kg

$CSL = F(ROP, D_L, \sigma_L)$

$= F(24931.49, 15000, 6037.39)$

$= 0.95$

$2 \times 150 \times 3 = 900$

$450 \times 12 =$

4.

(b)

$$\eta = \sqrt{\frac{\sum D h c}{2S}}$$

① Annual Ordering Cost $= 5000 \times 3 + n \times 450$

Annual Holding Cost $= \frac{Dhc}{2n} + \frac{Dhc}{2n} + \frac{Dhc}{2n} = \frac{3\,Dhc}{2n}$

Total Cost

$$15000 + 450n + \frac{3Dhc}{2n}$$

3S

$$450 - \frac{3}{2} Dhc \cdot \frac{1}{n^2} = 0$$

$$\boxed{2 \times 3S - 3Dhc}$$

$$3S = \frac{3}{2} \frac{Dhc}{n^2}$$

$$n^2 = \frac{3Dhc}{2 \times 3S}$$

$$n = \sqrt{\frac{3Dhc}{2 \times 3S}}$$

$$= \sqrt{\frac{\cancel{3} \times 15000 \times 0.1 \times 100}{2 \times \cancel{3} \times 150}}$$

$$= 7.071$$

$$Q = \frac{D}{n} = \frac{15000}{7.071} = 2121.32 \times 3 = \boxed{6363.96}$$

Common ordering Cost $= \frac{5000 \times 3}{7.071} = \$2121.34$

<u>2014-15</u>

4.
(a)

$DXL = \frac{500}{0.5} = 1000$ bar.

$L = \frac{1000}{730,00} \times 365 = 5$

transit time = 4 day.

(ii) transportation cost $= 73,000 \times 10$
$= 730,000$ tk

(iii) Holding cost $= 500 \times 0.2 \times 500 + \frac{2000}{2} \times 0.4 \times 500$
$= 150,000$ tk

5.
(a)

$D = 18$. $C = 60$, $S = 45$

$h = 0.25$

$Q = 390$

cycle inventory cost = Holding cost + Ordering cost

$CSL = (ROP, D_L, \sigma_L)$

$300,$

$\cancel{1.4} = \frac{ROP - 300}{30\sqrt{3}}$

$ROP =$

(b)

$ss = F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$= F_S^{-1}(0.9) \times \sqrt{2} \times 5$

$CSL = F(ROP, D_L, \sigma_L)$

$Rop = D \times L + ss$

$= 18 \times 2 + ss$

6. (a) $D = 100$, $\sigma_D = 30$, $L = 3$. $H = 9.4$. $S = 35$

$CSL = 0.92$

$ss = F^{-1}(CSL) \times \sqrt{3} \times 30$

$= 73$

$Rop = 100 \times 3 + ss$

$O^* = F^{-1}(CSL, \mu, \sigma)$

$= \cancel{143} \; \cancel{273}$

$= 374$ unit

$Rop = \cancel{90} \; 373$

Annual $Cf =$

$CSL =$

2010-11

3.

(b)

$\sigma_D = 100. \quad D = 200. \quad L = 3. \quad S_L = ?$

$SS = F_S^{-1}(CSL) \times \sigma_L$

$\sigma_L = \frac{300}{1.13} = 265.49$

$265.49^2 = 3 \times 100^2 + 200^2 \times S_L^2$

$S_L = 1$ day

$\boxed{D = 600}$

2011-12

1.

(a) $\sigma_L = \frac{2400}{F_S^{-1}(0.96)} =$

$L\,\sigma_D^2 + D^2 S_L^2 = \sigma_L^2$

3 1000 600

$$K = \frac{21900}{10} = 2190 \text{ unit}$$

$$\eta = \sqrt{\frac{2 \times 10{,}000 \times 0.2 \times 50}{2\,(500 + 125 \times 3)}} = 12.093$$

$$Q = \frac{D}{\eta} = \frac{10{,}000}{12.093} = 764$$

Total =

$$Q^{d} = \boxed{\frac{dD}{(C-d)h} + \frac{CQ^{2}}{C-d}}$$

Expected Cost = P (demand ≤ 1000) × average Cost

$= 0.1 \times 1 \times 5 = 0.5$

$= 0.25 \times 1 \times 5 = 1.25$

=

8a. Alternate supplier

Material Cost $= 52 \times 1000 \times 0.97 = 50440$

cycle inventory $= \frac{8000}{2} = 4000$

$SS = 1.64485 \times \sqrt{6 \times 300^2 + 4^2 \times 1000^2}$

$= 6689.5$

Total Cost $= 50440 + (4000 + 6689.5) \times 0.25 \times 0.97$

$= \$53032.2$

					Geometric Mean	N
1	1/3	1	1/3	1/3	0.517	0.095
3	1	2	1	1	1.43	0.262
1	1/2	1	1/2	1/2	0.659	0.12
3	1	2	1	1	1.43	0.262
3	1	2	1	1	1.43	0.262
Sum 11	23/6	8	23/6	23/6		

$\Sigma \lambda_x = 5.466$, $\lambda_{max} = 5.018$

$CI = \frac{\lambda_{max} - n}{n-1} = \frac{5.018-5}{5-1} = \frac{0.018}{4}$

$= 0.0045$

$CR = \frac{0.0045}{1.12} = 0.4\%$

Transportation   D = 120,000 yea.   C = 120

H = 120 × 0.25 = 30

L = 5+1 = 6 da, $\frac{20,000}{10} = 2000$

$Q = \frac{2000}{2} = 1000$

$\frac{L}{2}$ dr = $6/2 \times \frac{120,000}{360}$

= 292

In transit invt = $\frac{120,000}{365} \times 5$

= 1644

1010 + 950 + 144×360

Adding cost

= 3630 × 30

= 108900

2012-13

3.4668

4.

(a)

			G-M.	Nor
1	3	3	2.08008	0.6 ~~0.59999~~
$\frac{1}{3}$	1	1	0.69336	0.2
$\frac{1}{3}$	1	1	0.69336	0.2
Sum $5/3$	5	5		

$\lambda_{max} = \cancel{2.99998}$ 3

$C.I = \frac{\cancel{2.99998}\, 3 - 3}{2} = 0$

(b) Lead time = 3+1 = 4 days.

Holding Cost = (cycle Inventory + SS) × 0.2 × 10

$= \left(\frac{29200}{2} + 2400\right) \times 0.2 \times 10$

$= \$34,000$

Demand during lead time $= \frac{730,000 \times 4}{365} = 8000$

SS = 8000 × 0.3 = 2400

Ordering cost = $\frac{730,000}{29200} \times$

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ}{C-d}$$

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d}$$

5. (c)

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d}$$

$$= \frac{0.15 \times 120,000}{(3-0.15) \times 0.2} + \frac{3 \times 6324}{3-0.15}$$

$$= 38236 \text{ units}$$

Cycle Inventory $= \frac{Q^d}{2} = \frac{38236}{2} = 19118$

Average flow time $= \frac{Q^d}{2D} = \frac{38236}{2 \times 120,000} = 0.1593$

Forward Buy $= Q^d - Q$

$= 38236 - 6324$

$= 31912$ unit

6.

(c) $\mu = 2500$, $\sigma = 500$ $L = 2$, $T = 4$

$D_{TL} = 2500 * (4+2) = 15000$

$\sigma_{T+L} = \sqrt{6}\ 500 =$

$SS = F^{-1}(.90) \times \sqrt{6}\ 500 = 1568$

Order upto level $= 15000 + 1568$
$= 16568$ unit.

7.

(a) Truck capacity $= \frac{21900}{10} = 2190$ unit.

Order frequency $= \sqrt{\frac{3 \times 10{,}000 \times 0.2 \times 50}{2\,(500 + 125 \times 3)}}$

$= 13.09$

Order Quantity each $= \frac{10{,}000}{13.09} = 764$

Total order quantity $= 764 \times 3 = 2292$

Truck has capacity $= 2190$ unit

Now lot size per supplier $= \frac{2190}{3} = 730$ unit

number of order $= \frac{10{,}000}{730} = 13.7$

Ordering cost $= 13.7 \times 125$

Holding cost $= \frac{730}{2} \times 0.2 \times 50 =$

4. (b)

$D = 5000$. $C = 30$. $h = 0.2$; $S = 250$

$s = 50$. $K = 1398$

$$n = \sqrt{\frac{3 \times 5000 \times 0.2 \times 30}{2(250 + 50 \times 3)}} = 10.6$$

order quantity $= 3 \times \frac{5000}{10.6} = 1415$

Truck capacity $= 1398$

order qu. $= 1398$

$$Q^* = \frac{1398}{3} = 466$$

$$n = \frac{3 \times 5000}{1398} = 10.73$$

ordering cost per supplier $= 10.73 \times 50 + \frac{250 \times 10.73}{3}$

$= \$1430$

2014-15

4.
(b) $C = 500$, $D = 73{,}000$, $SS = 500$.

$R = \frac{500}{0.5} = 1000$

Demand per day $= \frac{73{,}000}{365} = 200$

Lead time $= \frac{1000}{200} = 5$

(i) transit time $= 5 - 1 = 4$ days.

(ii) transportation cost $= 730{,}000 \times 10$
$= 7300000$ tk

(iii) Holding cost $= \frac{2000}{2} \times 0.2 \times 500$
$= \$100{,}000$

On transit $= \frac{73000}{365} \times 4 = \boxed{800 \text{ unit}}$

6.
(b)

$D_E = 3000$, $S = 5$, $P = 30$, $C = 10$

$C_u = 30 - 10 = 20$

$C_o = 10 - 5 = 5$

$CSL = \frac{C_u}{C_o + C_u} = \frac{20}{25} = 0.8$

(ii) Marginal benefit $= P(\text{Demand} \geq 3000) \times C_u$

$= 0.6 \times 20 = 12$

Marginal loss $= P(\text{Dem} \leq 3000) \times C_o$

$= 0.4 \times 5 = 2$

$\boxed{\text{Marginal value} = 12 - 2 = 10}$